

# Composite materials: Modeling, Processing, and Characterization

Structural Modeling and Analysis

8-9 June 2021, 14:00-16:00

Speaker: Marco Petrolo, [www.mul2.com](http://www.mul2.com)

Contributors:

Prof. Erasmo Carrera, Dr. Ibrahim Kaleel, Dr. Manish H. Nagaraj

# Webinar Organization

- Structural models for composites, **Petrolo**, 8 June, 14:00–16:00
- Processing methods and simulation, **Zobeiry**, 8 June, 16:00–18:00
- Multiscale models and failure analysis, **Petrolo**, 9 June, 14:00–16:00
- Evolution of properties during processing and characterization methods, **Zobeiry**, 9 June, 16:00–18:00
- Fabrication process of Boeing 777x and 787, **Zobeiry**, 10 June, 16:00–18:00
- Safety factor, building block and substantiation of composite aircraft, **Zobeiry**, 11 June, 16:00–18:00

# The MUL2 Group



## MUL2 Staff

**Head of the Group**  
Professor Erasmo Carrera

### Associate Professors

Alfonso Pagani  
Marco Petrolo

### Assistant Professors

Matteo Filippi  
Enrico Zappino

### Post-Doc

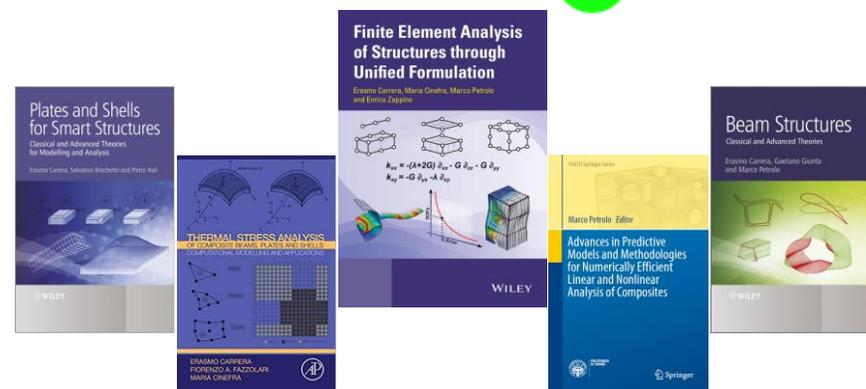
Riccardo Augello

### Current Projects

- PRIN - DEVISU
- H2020 - ERC - PRE ECO
- Clean Sky 2 - CASTLE
- Clean Sky 2 - Airgreen 2
- Embraer - Global/Local Analysis of composite wing Structures (GLAS) and Validation projects

### PhD Students

Rodolfo Azzara  
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### Research topics

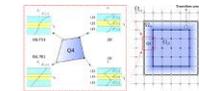
- Aeroelasticity and Fluid-structure Interaction
- Aircraft and Spacecraft Design
- Best Theory Diagrams and Asymptotic/Axiomatic Analysis
- Biomechanics
- Composites
- Damage and Failure
- Inflatable Structures
- Meshless methods
- Multifield Analysis
- Rotor-Dynamics
- Structure Mechanics and FEM
- Global/Local Methods and plug-in for Abaqus/Nastran

### Upcoming Events

- Sept 2021 - 26th Congress of the Italian Association of Aeronautics and Astronautics - [www.aidaa2021.it](http://www.aidaa2021.it)
- May 2022 - 10th Design, Modelling and Experiments of Advanced Structures and Systems Conference, DEMEASS2021 Scopello, Italy
- TBD 2022 - 3rd International Conference on Mechanics of Advanced Materials and Structures, ICMAMS2021, College Station, TX, USA
- Sept 2024 - 34th International Council of the Aeronautical Sciences, ICAS2024, Florence, Italy

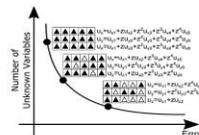
### Node-dependent kinematics

E. Zappino, G. Li, A. Pagani, E. Carrera, A.G. de Miguel  
*Use of higher-order Legendre polynomials for multilayered plate elements with node-dependent kinematics*  
Composite Structures, 2018



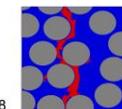
### Best Theory Diagrams

E. Carrera, M. Petrolo  
*Axiomatic/Asymptotic Method and Best Theory Diagram for Composite Plates and Shells*  
Encyclopedia of Continuum Mechanics, Springer, 2018



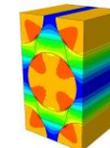
### Progressive failure

I. Kaleel, M. Petrolo, A.M. Waas, E. Carrera  
*Micromechanical Progressive failure Analysis of Fiber-Reinforced Composite Using Refined Beam Models*  
Journal of Applied Mechanics, 2018

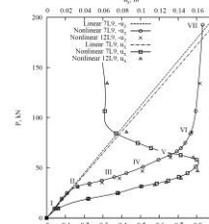


### Micromechanics

A.G de Miguel, A. Pagani, W. Yu, E. Carrera  
*Micromechanics of periodically heterogeneous materials using higher-order beam theories and the mechanics of structure genome*  
Composite Structures, 2017

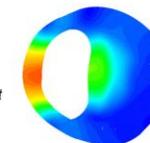


### Non-linearity



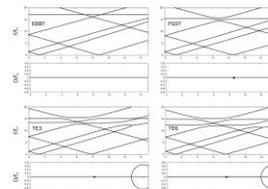
### Biomechanics

E. Carrera, D. Guarnera, A. Pagani  
*Static and free-vibration analysis of dental prosthesis and atherosclerotic human artery by refined finite element models*  
Biomechanical Modeling and Mechanobiology, 2018.



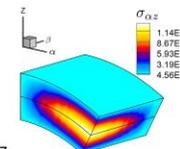
A. Pagani, E. Carrera  
*Unified formulation of geometrically nonlinear refined beam theories*  
Mechanics of Advanced Materials and Structures, 2016

### Rotordynamics



### Composites and multifield

M. Cinefra, M. Petrolo, G. Li, E. Carrera  
*Variable kinematic shell elements for composite laminates accounting for hygrothermal effects*  
Journal of Thermal Stresses, 2017



### Global/local analysis and plug-in for Abaqus/Nastran



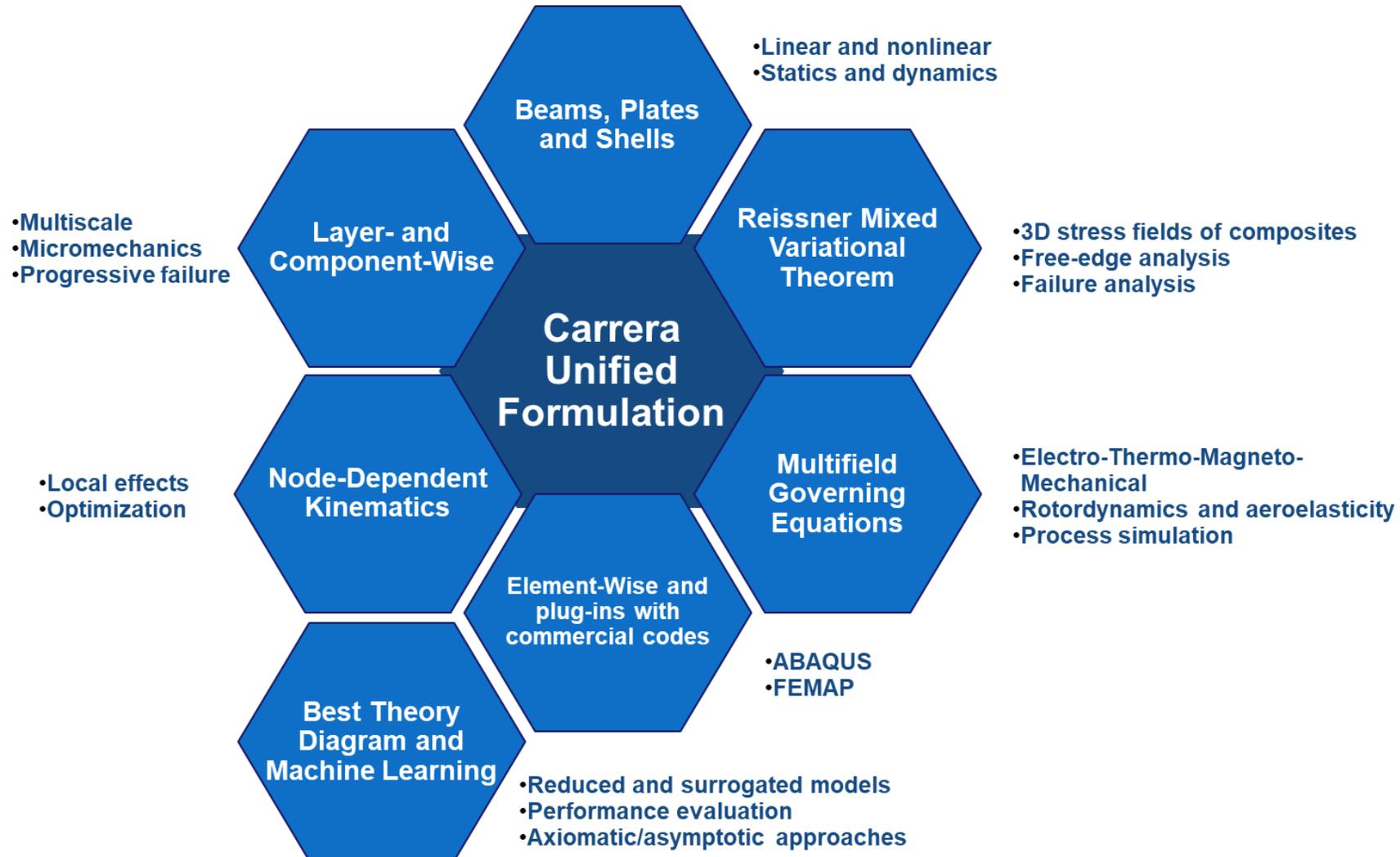
E. Carrera, M. Filippi  
*Variable kinematic one-dimensional finite elements for the analysis of rotors made of composite materials*  
Journal of Engineering for Gas Turbines and Power, 2014

## Network and Partners:

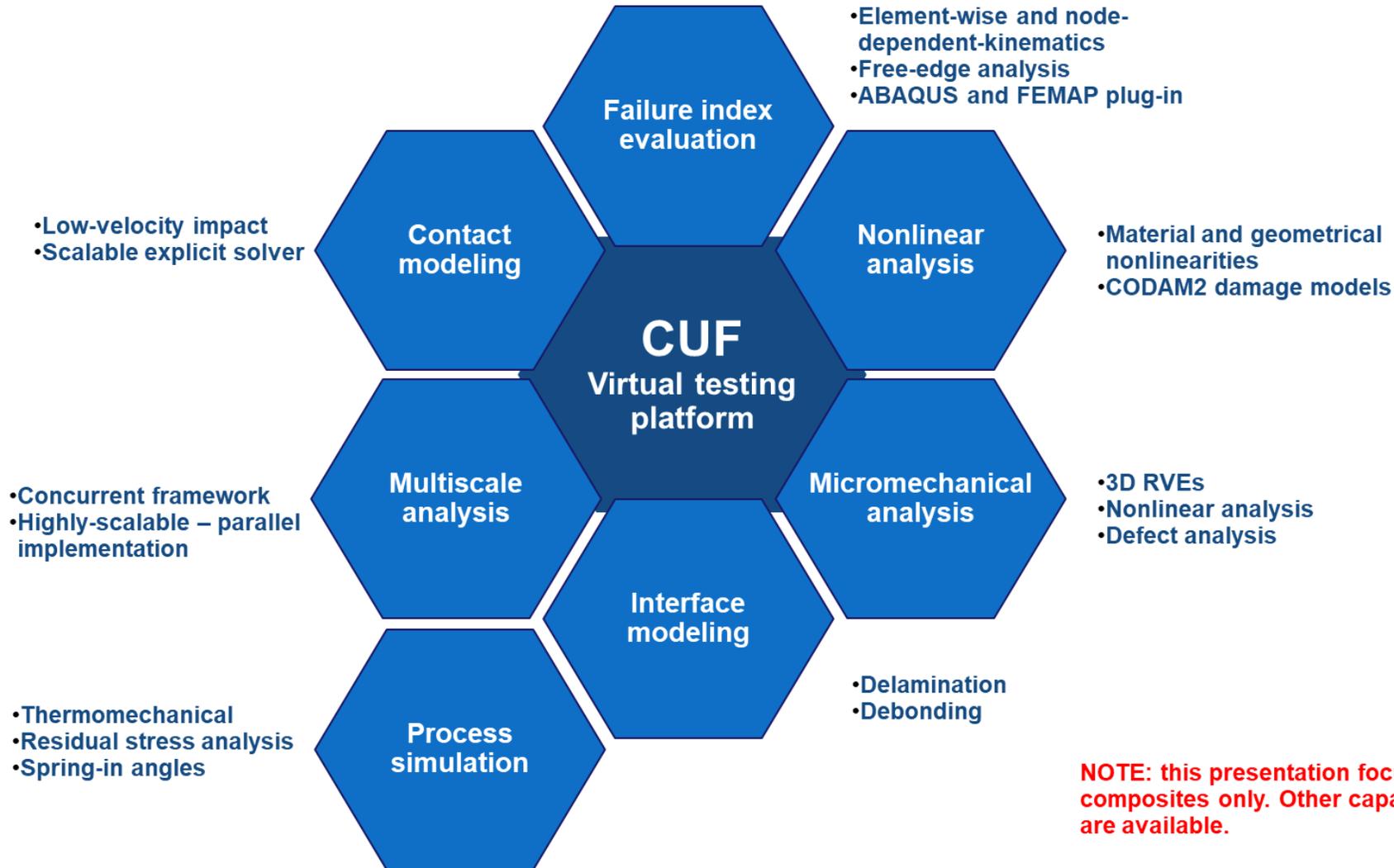


# MUL2 – Theories of Structures and FEM

MUL2 is an in-house FE code entirely based on the Carrera Unified Formulation (CUF)



# MUL2 – Current Capabilities for Composites



# Outline

- Introduction
  - PART I – Structural Theories
- Brief Overview of Theories for Metallic Structures
- Theories for Composite Structures
- Numerical Examples
  - PART II – Failure and Multiscale Analyses
- Failure Onset and Progression
- Micromechanics
- Multiscale

# INTRODUCTION

# Aims 1/2

- Available **options** for the structural analysis of **metallic and composite structures** concerning the choice of **1D, 2D, and 3D theories**
- Proper **modeling of** various types of **mechanical behaviors** and the associated **solution's efficiency**
- Multiple **problems** are considered, including **linear and nonlinear** analyses and **static and dynamic** settings
- **Guidelines on the proper selection of a model** are outlined, and **quantitative estimations on the accuracy** are provided
- Not many theoretical details, but **warnings, hints and references** for further study

# Aims 2/2

Problem	HOST DOF	3D DOF	HOST/3D	Error	Variable and Convergence Pattern
<b>Non-homogeneous atherosclerotic plaque - Beam</b>	23529	761244	3%	3%	Transverse axial strain, HOST -> 3D
<b>Composite C-section spar - Beam</b>	89175	2276739	4%	25%	Free-edge peeling stress, 3D -> HOST
<b>Composite Omega stringer - Beam</b>	142476	4560150	3%	30%	Free-edge failure index, 3D -> HOST
<b>Composite notched specimen - Plate</b>	10000	10000000	0.1%	3%	Tensile peak stress, HOST -> 3D
<b>Multilayered beam - Beam</b>	23595	63210	37%	0.4%	Plastic strain, HOST -> 3D
<b>Double-swept blade - Beam</b>	13200	203808	6%	1%	Natural frequencies, HOST -> 3D
<b>Viscoelastic beam - Beam</b>	5475	56400	10%	5%	Modal loss factor, HOST -> 3D
<b>Randomly distributed RVE - Beam</b>	13642	31524	43%	2%	Local shear strain, HOST -> 3D
<b>Lattice structure - Beam</b>	13584	617580	2%	1%	Displacement, HOST -> 3D
<b>Three-point bending of a sandwich beam - Beam</b>	14229	201504	1%	0%	Transverse stress, HOST -> 3D
<b>Low-velocity impact on a bi-metallic plate - Plate</b>	10659	856251	1%	16%	Plastic strain, 3D -> HOST
<b>Large deflections in asymmetric cross-ply beams - Beam</b>	5124	573675	1%	7%	Shear stress, HOST -> 3D
<b>Disbonding in sandwich beams - Beam</b>	41160	171888	24%	1%	Peak load, HOST -> 3D
<b>Curing of a composite part - Beam</b>	16569	599571	3%	0%	Spring-in angle, 3D -> HOST

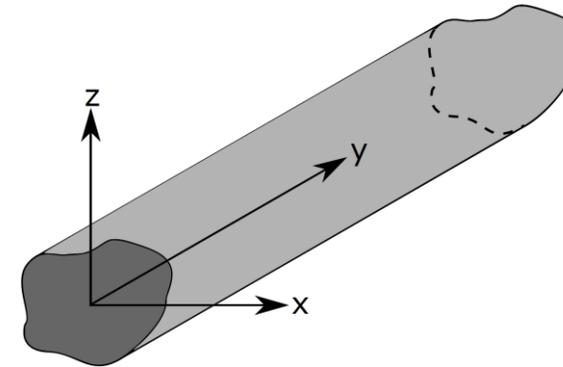
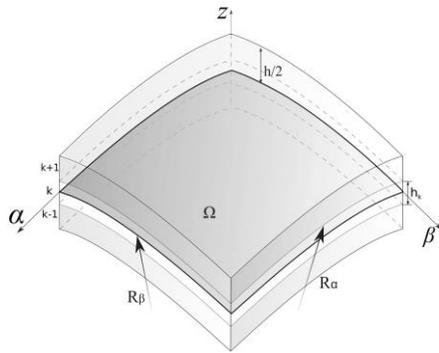
Carrera E. et al, (2021), COMPOSITE STRUCTURES, Vol. 264, DOI:10.1016/j.compstruct.2021.113671

# Basic Nomenclature

- 1D, 2D and 3D theories: **beams, plates** and **shells, solids**
- **Classical theories:** Euler–Bernoulli, Timoshenko, Kirchhoff, Reissner–Mindlin, Classical Lamination Theory (CLT), First–Order Shear Deformation Theory (FSDT)
- **Refined or higher–order theories:** any theory beyond classical ones, e.g., Third–Order Shear Deformation Theory (TSDT)
- **Degrees of Freedom (DOF):** number of unknown variables, e.g, **Timoshenko** has **five DOF per node**; in **commercial codes**, each node can have up to **six DOF**

# A Definition for *Structural Theory*

- A **structural theory** is a set of mathematical relations and assumptions to **reduce/approximate the 3D problem to 2D or 1D ones**
- **2D theory**: the approximation may be seen as a **through-the-thickness integration**; assumptions are made concerning the **through-the-thickness behavior** of the structure, e.g., unstretchable normal; **unknowns** over the **in-plane coordinates**



- **1D theory**: the approximation is an **integral over the cross-section**; assumptions are made concerning the **cross-sectional behavior** of the structure, e.g., no shear deformability; **unknowns** over the **axial coordinate**

# PART I – Structural Theories

## BRIEF OVERVIEW OF THEORIES FOR METALLIC STRUCTURES

# Classical Theories 1/2

- Notes:
  - For the sake of brevity, the **focus is on 2D models**, namely, plates and shells; in most cases, the **assumptions** used to develop such 2D models can be **applied to 1D models as well**
  - For **shell** theories, the standard notation is based on **curvilinear coordinates** and displacements are  $u_\alpha$ ,  $u_\beta$ , and  $u_z$ ; for **plates** and **beams**, the components are  $u_x$ ,  $u_y$ , and  $u_z$

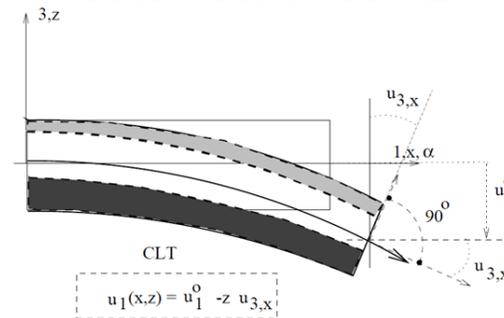
- The **simplest 2D theory** is the **Kirchhoff-Love** (or **Euler-Bernoulli** for 1D):

$$u_x = u_{x1} - z u_{z1,x}$$

$$u_y = u_{y1} - z u_{z1,y}$$

$$u_z = u_{z1}$$

- 3 DOF**



- Assumptions: **no transverse shear stress, no normal stretching**
- The **derivatives** of the transverse displacement have the physical meaning of **rotations of the normals**

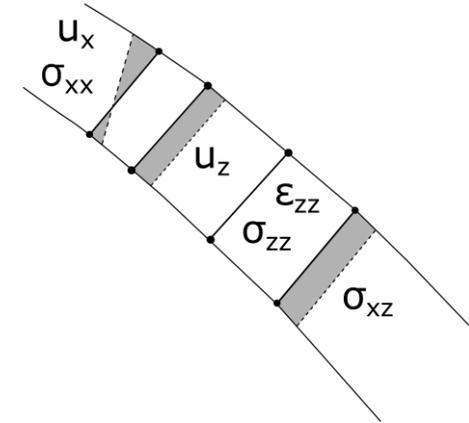
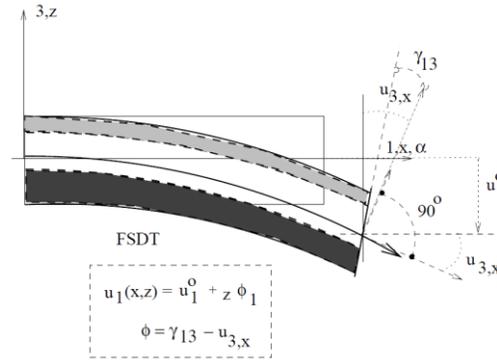
# Classical Theories 2/2

- The First Order Shear Deformation Theory (**FSDT**) assumes a **constant shear distribution along the thickness**

$$u_x = u_{x1} + zu_{x2}$$

$$u_y = u_{y1} + zu_{y2}$$

$$u_z = u_{z1}$$



- 5 DOF**
- The **additional two terms** are rotations of the normals corrected by **shear**
- Stress **in-plane components** can be **linear**, the **transverse shear** stress is **constant**, and the **normal stretching and stress null**
- The accuracy for global responses **improved via shear correction factors**. Such factors are **problem-dependent** as they depend on the distribution of shear stresses

# A Simple Case with Normal Stretching

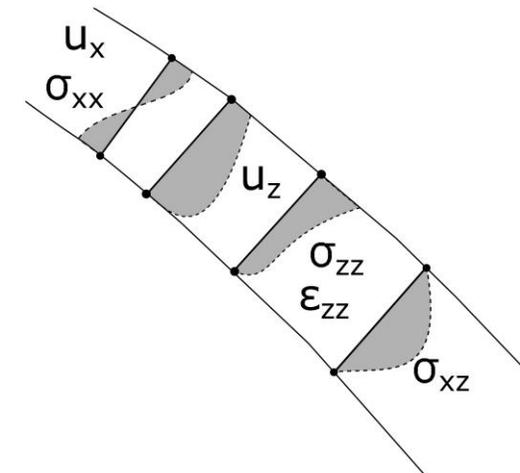
- Let us consider a **3D temperature distribution** over an **isotropic plate** with a constant distribution:  $T(x, y, z) = T_0$
- Being  $\alpha$  the coefficient of thermal expansion, the **normal strains induced** by the heat distribution are  $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \alpha T_0$
- Assuming linear geometrical relations:  $\varepsilon_{zz} = u_{z,z}$
- Integrating:  $u_z = u_{z1} + \alpha T_0 z$
- A plate theory with **at least a linear transverse displacement field** is necessary to capture the **normal transverse strain** caused by the **constant distribution of temperature** across the thickness
- With a similar procedure, the **linear distribution of temperature** leads to a **quadratic distribution of transverse displacement**
- In the most general case,  $T$  stems from a **heat-conduction problem** and, in the case of **thick multilayered structures**, may assume a **layer-wise nonlinear distribution** and require **higher-order layer-wise displacement fields**

E. Carrera: Journal of Thermal Stresses, 23(9):797-831, 2000; AIAA Journal, 40(9):1885-1896, 2002; AIAA Journal, 43(10):2232-2242, 2005

# Higher-Order Structural Theories (HOST)

- The **enrichment of the displacement field** is a common approach to **remove assumptions** from structural theories
- A **2D or 1D** model based on an **infinite expansion** would guarantee the **exact 3D solution**

$u_x = u_{x1} + zu_{x2} + z^2u_{x3} + z^3u_{x4}$ $u_y = u_{y1} + zu_{y2} + z^2u_{y3} + z^3u_{y4}$ $u_z = u_{z1}$	$u_x = u_{x1} + zu_{x2} + z^2u_{x3} + z^3u_{x4}$ $u_y = u_{y1} + zu_{y2} + z^2u_{y3} + z^3u_{y4}$ $u_z = u_{z1} + zu_{z2} + z^2u_{z3} + z^3u_{z4}$
$u_x = u_{x1} + zu_{x2}$ $u_y = u_{y1} + zu_{y2}$ $u_z = u_{z1} + zu_{z2} + z^2u_{z3}$	$u_x = u_{x1} + zu_{x2} + \sin\left(\frac{z\pi}{h}\right)u_{x3} + e^{z/h}u_{x4}$ $u_y = u_{y1} + zu_{y2} + \sin\left(\frac{z\pi}{h}\right)u_{y3} + e^{z/h}u_{y4}$ $u_x = u_{z1} + zu_{z2} + \sin\left(\frac{z\pi}{h}\right)u_{z3} + e^{z/h}u_{z4}$

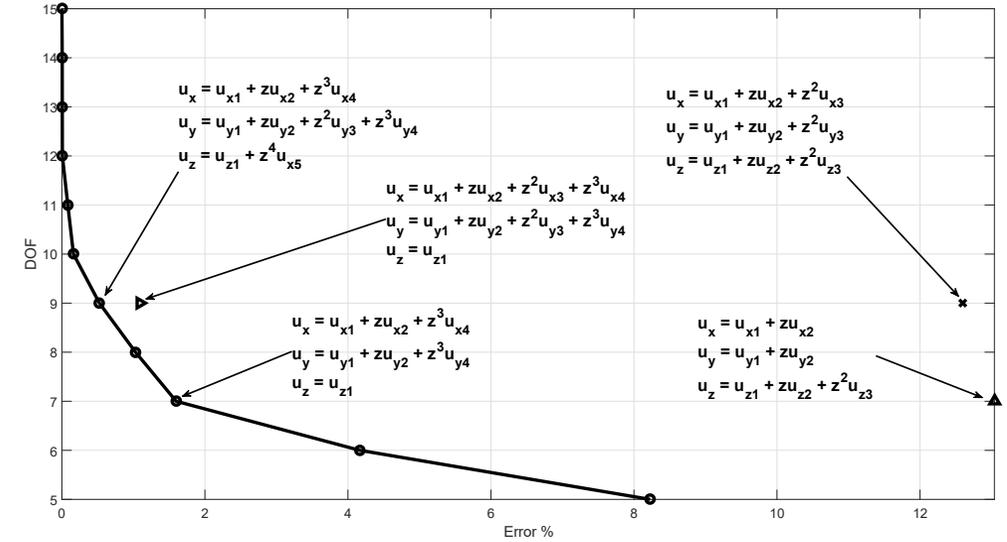
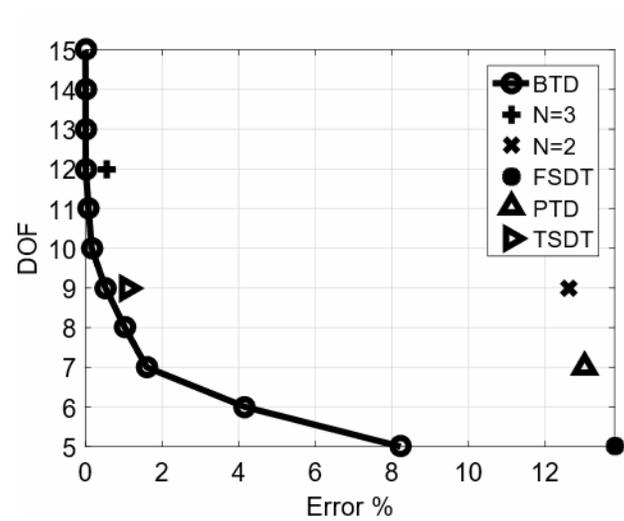


J.N. Reddy, International Journal of Engineering Science, 23(3):319 -330, 1985

K. Washizu, Variational Methods in Elasticity and Plasticity. Pergamon, Oxford, 1968

# HOST's Accuracy is Problem Dependent

- S-S 0/90/0 Shell, Free Vibrations, 10 Modes,  $\alpha/h = 10$
- The **solid line is the Best Theory Diagram**, i.e., the set of shell theories **minimizing DOF** per node and **maximizing the accuracy**.
- The error distribution is problem-dependent
- **Carrera Unified Formulation**: framework to obtain any theory



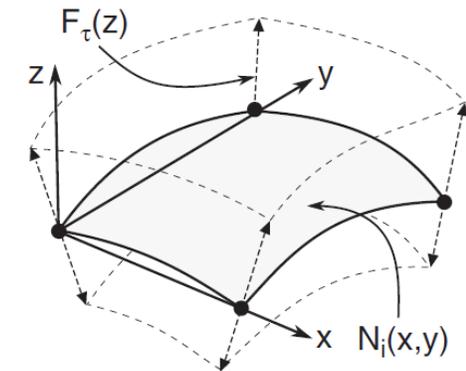
Petrolo M., Carrera E., (2020), ACTA MECHANICA, Vol. 231, pp. 395-434, DOI: 10.1007/s00707-019-02601-w

# Carrera Unified Formulation (CUF)

$u_x =$	$u_{x1} +$	$zu_{x2} +$	$z^2u_{x3} +$	$z^3u_{x4} +$	$\dots$
$u_y =$	$u_{y1} +$	$zu_{y2} +$	$z^2u_{y3} +$	$z^3u_{y4} +$	$\dots$
$u_z =$	$u_{z1} +$	$zu_{z2} +$	$z^2u_{z3} +$	$z^3u_{z4} +$	$\dots$
	$N = 0$	$N = 1$	$N = 2$	$N = 3$	$N = \dots$
	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = \dots$
	3 DOF	3 DOF	3 DOF	3 DOF	$\dots$

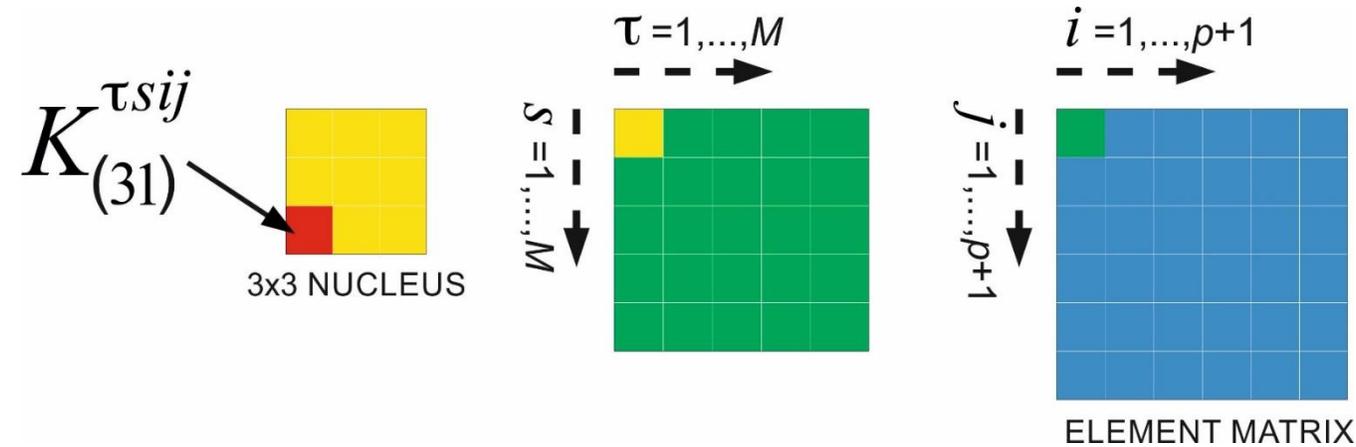
**CUF Displacement Field:**  $u(x, y, z) = F_\tau(z)u_\tau(x, y)$

**FEM:**  $u = F_\tau N_i u_{\tau i}$



Carrera E., Archives of Computational Methods in Engineering (2003), Vol. 10, 3, pp. 215–296

# CUF – Fundamental Nucleus of Stiffness Matrix



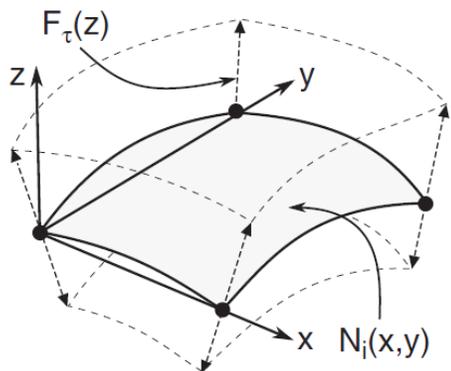
$$K_{xx}^{\tau i s j} = \int_{\Omega} N_i N_j d\Omega \int_A C_{55} F_{\tau,z}^i F_{s,z}^j dz + \int_{\Omega} N_{i,y} N_{j,y} d\Omega \int_A C_{66} F_{\tau}^i F_s^j dz + \int_{\Omega} N_{i,x} N_{j,y} d\Omega \int_A C_{16} F_{\tau}^i F_s^j dz +$$

$$+ \int_{\Omega} N_{i,y} N_{j,x} d\Omega \int_A C_{16} F_{\tau}^i F_s^j dz + \int_{\Omega} N_{i,x} N_{j,x} d\Omega \int_A C_{11} F_{\tau}^i F_s^j dz$$

$$K_{xy}^{\tau i s j} = \int_{\Omega} N_i N_j d\Omega \int_A C_{45} F_{\tau,z}^i F_{s,z}^j dz + \int_{\Omega} N_{i,y} N_{j,y} d\Omega \int_A C_{26} F_{\tau}^i F_s^j dz + \int_{\Omega} N_{i,x} N_{j,y} d\Omega \int_A C_{12} F_{\tau}^i F_s^j dz +$$

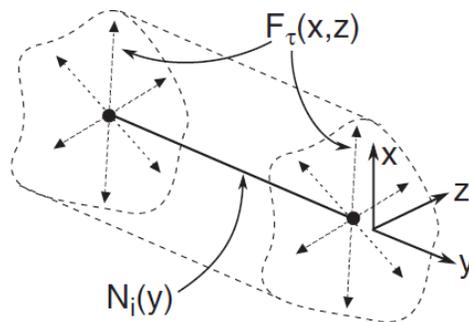
$$+ \int_{\Omega} N_{i,y} N_{j,x} d\Omega \int_A C_{66} F_{\tau}^i F_s^j dz + \int_{\Omega} N_{i,x} N_{j,x} d\Omega \int_A C_{16} F_{\tau}^i F_s^j dz$$

# CUF – 1D, 2D, 3D



$$\begin{aligned}
 u_x &= u_{x1} + zu_{x2} + z^2u_{x3} + z^3u_{x4} + \dots \\
 u_y &= u_{y1} + zu_{y2} + z^2u_{y3} + z^3u_{y4} + \dots \\
 u_z &= u_{z1} + zu_{z2} + z^2u_{z3} + z^3u_{z4} + \dots
 \end{aligned}$$

$$\begin{aligned}
 u_x &= u_{x1} + xu_{x2} + zu_{x3} + x^2u_{x4} + xzu_{x5} + \dots \\
 u_y &= u_{y1} + xu_{y2} + zu_{y3} + x^2u_{y4} + xzu_{y5} + \dots \\
 u_z &= u_{z1} + xu_{z2} + zu_{z3} + x^2u_{z4} + xzu_{z5} + \dots
 \end{aligned}$$



**Equilibrium equations in Strong Form**  $\rightarrow \delta L_i = \int_V \delta u k u dV + \int_S \dots dS$

$$\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix}$$

$k_{xx} = -(\lambda + 2G) \partial_{xx} - G \partial_{zz} - G \partial_{yy}$   
 $k_{xy} = -\lambda \partial_{xy} - G \partial_{yx}$   
 $k_{xz} = \dots$   
 $\lambda = (E\nu) / [(1+\nu)(1-2\nu)]; \quad G = E / [2(1+\nu)]$

The diagonal (e.g.  $k_{xx}$ ) and the non-diagonal (e.g.  $k_{xy}$ ) terms can be obtained through proper index permutations.

**3D FEM Formulation**  $\rightarrow \delta L_i = \delta u_j k^{ij} u_i$

$$\begin{aligned}
 k_{xx}^{ij} &= (\lambda + 2G) \int_V N_{j,x} N_{i,x} dV + G \int_V N_{j,x} N_{i,z} dV + G \int_V N_{j,y} N_{i,y} dV; \\
 k_{xy}^{ij} &= \lambda \int_V N_{j,y} N_{i,x} dV + G \int_V N_{j,x} N_{i,y} dV
 \end{aligned}$$

$u = N_i(x, y, z) u_i$   
 $\delta u = N_j(x, y, z) \delta u_j$

**2D FEM Formulation**  $\rightarrow \delta L_i = \delta u_{sj} k^{\tau sij} u_{\tau i}$

$$\begin{aligned}
 k_{xx}^{\tau sij} &= (\lambda + 2G) \int_{\Omega} N_{i,x} N_{j,x} d\Omega \int_h F_{\tau} F_s dz + \\
 &+ G \int_{\Omega} N_{i,z} N_{j,z} d\Omega \int_h F_{\tau,z} F_{s,z} dz + G \int_V N_{i,y} N_{j,y} d\Omega \int_h F_{\tau} F_s dz; \\
 k_{xy}^{\tau sij} &= \lambda \int_{\Omega} N_{i,y} N_{j,y} d\Omega \int_h F_{\tau} F_s dz + G \int_{\Omega} N_{i,x} N_{j,y} d\Omega \int_h F_{\tau} F_s dz
 \end{aligned}$$

$u = N_i(x, y) F_{\tau}(z) u_{\tau i}$   
 $\delta u = N_j(x, y) F_s(z) \delta u_{sj}$

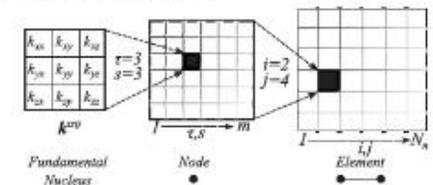
**1D FEM Formulation**  $\rightarrow \delta L_i = \delta u_{sj} k^{\tau sij} u_{\tau i}$

$$\begin{aligned}
 k_{xx}^{\tau sij} &= (\lambda + 2G) \int_l N_i N_j dy \int_A F_{\tau,x} F_{s,x} dA \\
 &+ G \int_l N_i N_j dy \int_A F_{\tau,z} F_{s,z} dA + G \int_l N_{i,y} N_{j,y} dy \int_A F_{\tau} F_s dA; \\
 k_{xy}^{\tau sij} &= \lambda \int_l N_{i,y} N_{j,y} dy \int_A F_{\tau} F_{s,x} dA + G \int_l N_i N_{j,y} dy \int_A F_{\tau,x} F_s dA
 \end{aligned}$$

$u = N_i(y) F_{\tau}(x, z) u_{\tau i}$   
 $\delta u = N_j(y) F_s(x, z) \delta u_{sj}$

CUF leads to the automatic implementation of any theory of structures through 4 loops (i.e. 4 indexes):

- $\tau$  and  $s$  deal with the functions that approximate the displacement field and its virtual variation along the plate/shell thickness ( $F_{\tau}(z)$ ,  $F_s(z)$ ) or over the beam cross-section ( $F_{\tau}(x, z)$ ,  $F_s(x, z)$ );
- $i$  and  $j$  deal with the shape functions of the FE model. (3D:  $N_i(x, y, z)$ ,  $N_j(x, y, z)$ ; 2D:  $N_i(x, y)$ ,  $N_j(x, y)$ ; 1D:  $N_i(y)$ ,  $N_j(y)$ ).



# Error of lower- and higher-order models for various applications involving isotropic structures [1]

Problem	Variable	Lower-order	Higher-order
<b>Thermo-mechanical, linear temperature profile, plate with <math>a/h = 10</math> [2]</b>	Transverse displacement	FSDT, 37 %	N = 2, 0.3 %
<b>Elastoplastic, bending of a bimetallic cantilever beam [3]</b>	Plastic strain	N = 2, 30 %	LE9, 6 %
<b>Dynamic response of a thin-walled cylinder [4]</b>	Transverse displacement	TBT, 80 %	N = 7, 5 %
<b>Wave propagation in a thin plate [5]</b>	Transverse displacement	N = 1, 30 %	N = 4, 0.1 %

[1] E. Carrera et al. Composite Structures, 264, 2021

[2] E. Carrera. AIAA Journal, 40(9):1885-1896, 2002

[3] I. Kaleel et al. In: Advances in Predictive Models and Methodologies for Numerically Efficient Linear and Nonlinear Analysis of Composites, Springer, 2019

[4] E. Carrera and A. Varello. Journal of Sound and Vibration, 331(24):5268-5282, 2012

[5] A.G. de Miguel et al. Journal of Sound and Vibration, 457:139-155, 2019

# PART I – Structural Theories

## THEORIES FOR COMPOSITE STRUCTURES

# Challenging Features of Composites 1/4

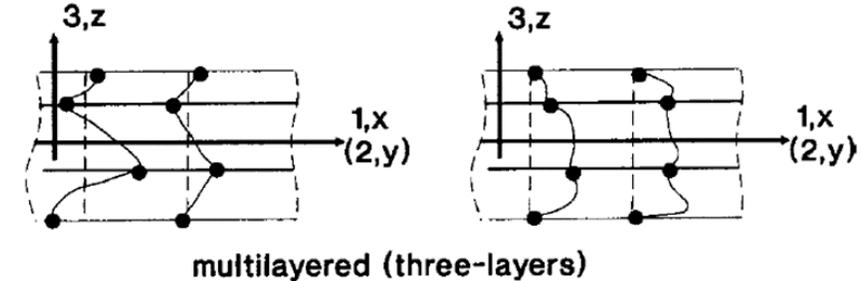
- In-plane anisotropy: **properties** of each layer can change significantly **depending on directions**
- The anisotropy may coexist with **high transverse shear and normal flexibility** and **couplings** between axial and shear strains. Further couplings can arise between in-plane and out-of-plane strains, e.g., in the case of non-symmetric stacking sequences
- Transverse anisotropy: **layer-wise discontinuous physical properties** along the thickness leading to compatibility and equilibrium conditions challenging to model
- The compatibility requires the **discontinuity of the slopes of displacement components** at the layer interfaces, i.e., the **zig-zag shape of the displacement field**. Such a zig-zag shape must coexist with the equilibrium conditions dictating the **continuity of the transverse stresses** at the interfaces, i.e., transverse shear and axial stresses must be interlaminar continuous. The literature refers to the sum of the two conditions as to the  **$C_0^Z$  requirements**
- Further challenges may stem from many scenarios: the analysis of **residual stress from the curing process**, the analysis of **damaged structures**, the presence of singularities such as **free edges**, the use of tens of layers, and the use of **sandwich structures with soft cores**

# Challenging Features of Composites 2/4

- High transverse deformability:
  - Common values for isotropic cases:  $\frac{E_L}{E_T} = 1, \frac{E}{G} = 2.6$ , with  $\nu = 0.3$
  - Common values for an orthotropic layer:

$$5 < \frac{E_L}{E_T} < 40, 2 < \frac{E_T}{G_{LT}} < 5, 10 < \frac{E_L}{G_{LT}} < 200$$

- Transverse displacement and stress:
- Koiter's recommendation:



“A refinement of the Love’s first approximation theory is indeed meaningless, in general, unless the **effects of transverse shear and normal stresses** are taken into account at same time”

Koiter WT, Proc. Of Symp. On the Theory of Thin Elastic Shells, Amsterdam, 1959

Cicala P, “Sulla teoria elastica della parete sottile”, Giornale del Genio Civile, 6e(9), 1959

Goldenveizer AL, “Theory of Thin Elastic Shells”, Pergamon Press, 1961

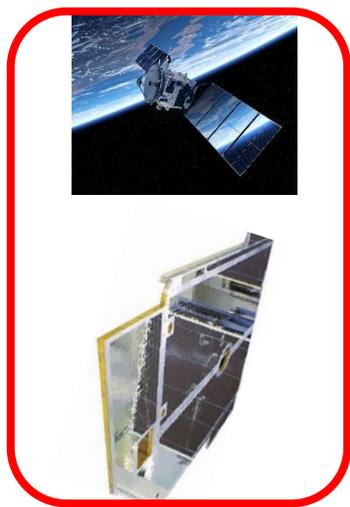
# Challenging Features of Composites 3/4

- **Koiter's** recommendation is **not enough** as **zigzag effects**, and **interlaminar continuity** are **additional requirements**
- The **coupling** between **in-plane** and **out-of-plane** normal **stresses** makes the fulfillment of the  $\sigma_{zz}$  **interlaminar continuity** particularly **difficult**
- A theory's **accuracy depends on multiple factors**, e.g., the **thickness** and **orthotropic ratios**, the **number of layers, laminations**, and the **outputs**, e.g., **global** ones such as **natural frequencies**, or **local** ones, such as **through-the-thickness stress** distributions

E. Carrera, Journal of Applied Mechanics, 66(4):1004-1012, 12 1999; E. Carrera, Journal of Sound and Vibration, 225(5):803- 829, 1999.

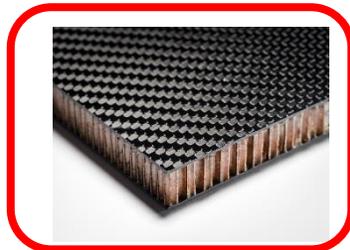
E. Carrera, Composite Structures, 50(2):183-198, 2000; A.G. de Miguel et al, Composites Part B: Engineering, 168:375-386, 2019.

# Challenging Features of Composites 4/4

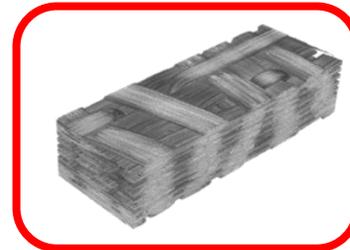


**Macro Structure** Macro-scale  $10^1$  m

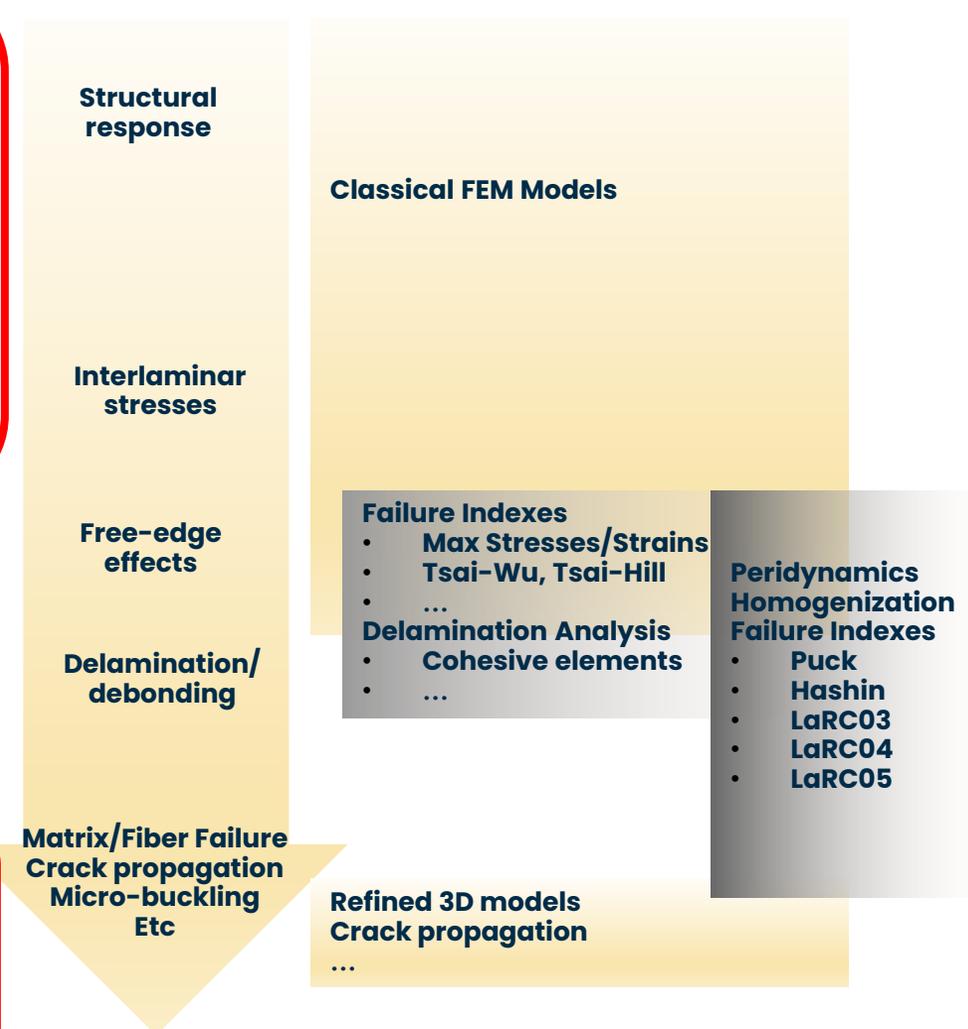
**Structural Component** Macro-scale  $10^0$  m



**Lamina & Layer** Meso-scale  $10^{-2}$  m

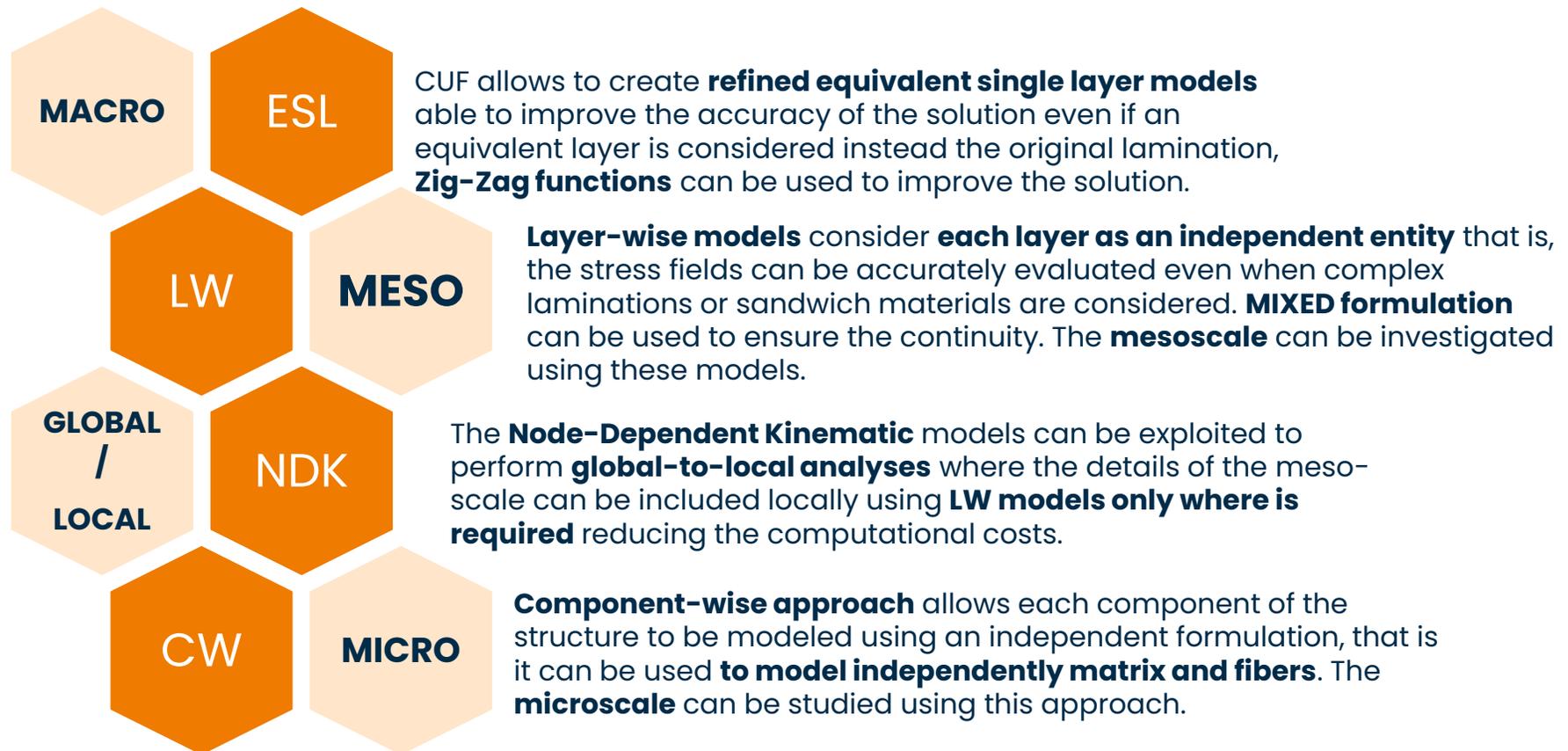


**Matrix & Fibre** Micro-scale  $10^{-6}$  m



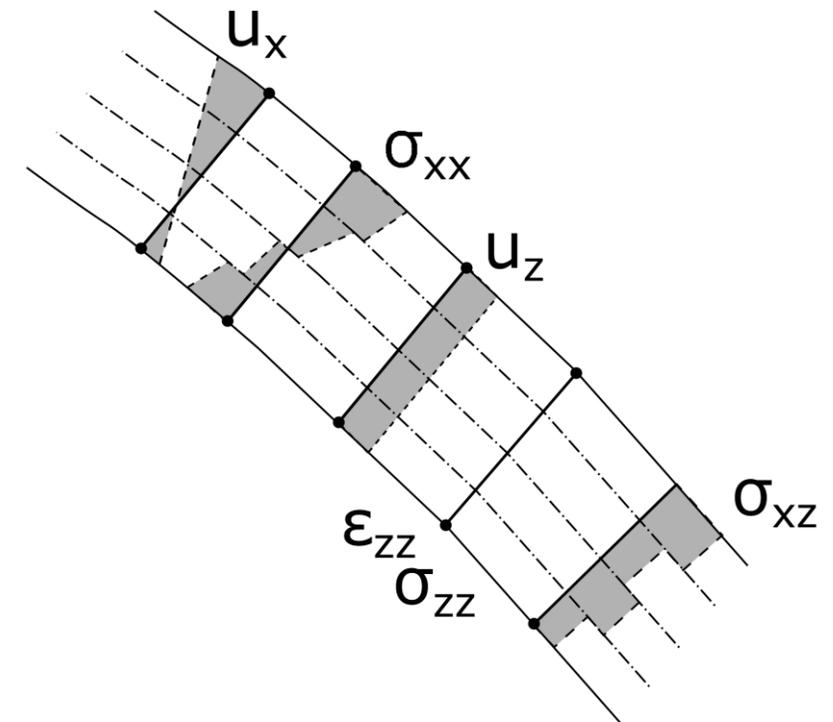
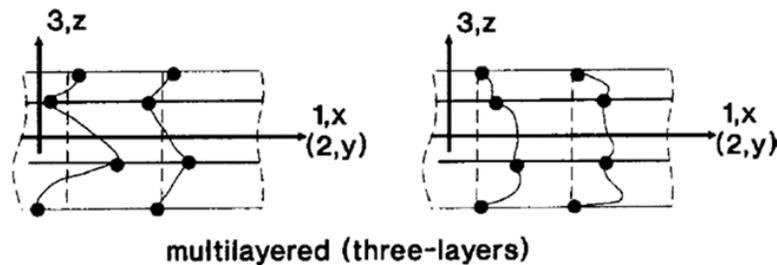
# Overview of Modeling Approaches

CUF provides models for all scales, including 1D, 2D, and 3D models



# Classical Theories – FSDT 1/2

- As in the metallic case, **in-plane** displacement and stress are usually **well-captured**
- The **transverse shear stress fails** to fulfill the top/bottom and interlaminar boundary conditions

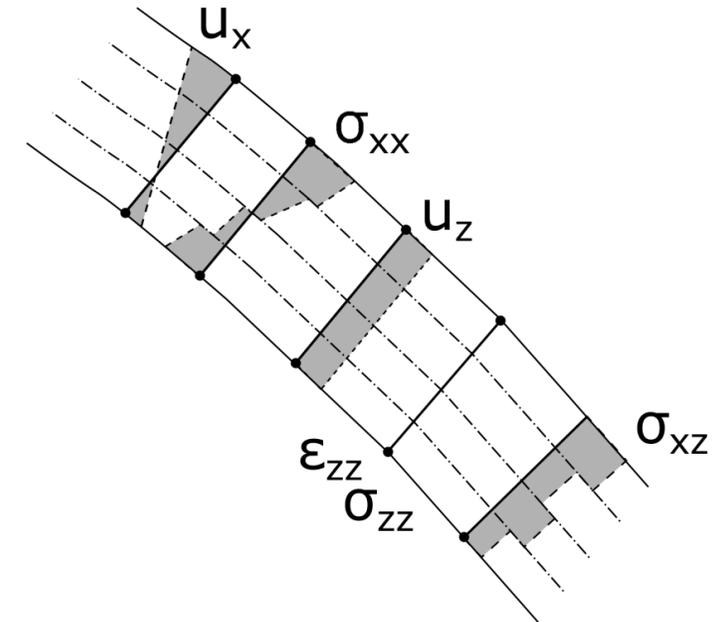


# Classical Theories – FSDT 2/2

- **Transverse normal** stresses/strains are **neglected**
- **Transverse shear** strains are considered. There are retained **constant** within the thickness direction
- **Transverse stresses** are **discontinuous** at the interface.
- **Koiter's** recommendation is **not considered**

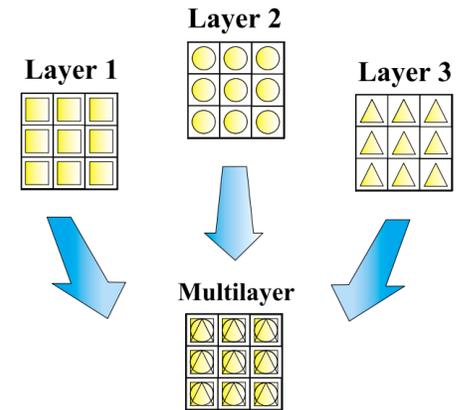
Consequently:

- FSDT works **well for global response** of **thin and moderately thin** laminated structures
- **No thick** laminated structures
- **No local phenomena**
- Shear **correction factors** can help but highly **problem-dependent**



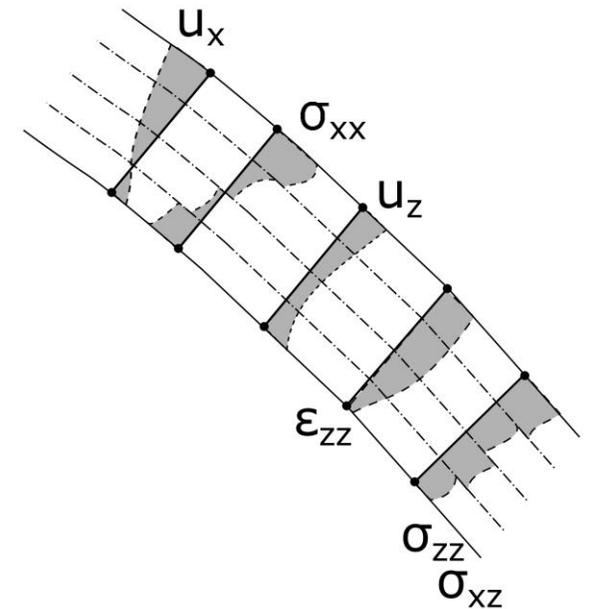
# HOST and Equivalent Single Layer (ESL)

- **ESL** is a modeling approach commonly used for CLT, FSDT and HOST
- **ESL** assumes a distribution of variables for the **whole laminate**
- **ESL homogenizes** the **mesoscale properties** (each layer) into **a single macroscale layer**



The **same HOST seen before** may be applied to composites:

- The **inclusion of transverse deformability** is decisive in many cases
- **Further refinements are necessary** as, for instance, the **interlaminar continuity** of transverse stresses is **not achieved**



# Zig-Zag Models

- **Aim:** improve the  $C_0^z$  requirements, i.e., **discontinuity** of the slopes of displacement components at the layer interfaces, and the continuity of the transverse stresses at the interfaces while **maintaining total DOF independent of the number of layers**
- There exist three ZZ classes [1]: **Lekhnitskii–Ren, Ambartsumian–Whitney–Rath–Das; Reissner–Murakami–Carrera**

Examples on the right: **FSDT and TSDT with Murakami ZZ**

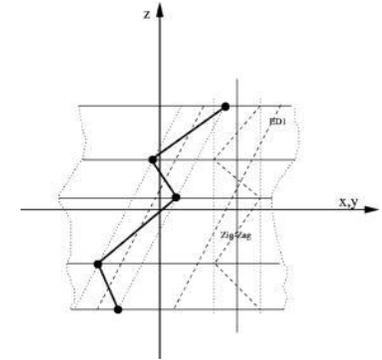
- $u_{xZ}, u_{yZ}, u_{zZ}$  are **additional variables** independent of layers
- The **exponent k** changes the sign of the zig-zag term in each layer to **obtain the discontinuity of the first derivative of the displacement variables** in the z-directions and due to transverse anisotropy

[1] Carrera E., Applied Mechanics Reviews (2003), 56, pp. 287–308

$$u_x = u_{x1} + zu_{x2} + (-1)^k \frac{2z_k}{h_k} u_{xZ}$$

$$u_y = u_{y1} + zu_{y2} + (-1)^k \frac{2z_k}{h_k} u_{yZ}$$

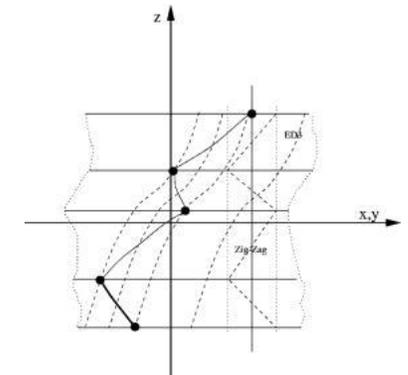
$$u_z = u_{z1}$$



$$u_x = u_{x1} + zu_{x2} + z^2u_{x3} + z^3u_{x4} + (-1)^k \frac{2z_k}{h_k} u_{xZ}$$

$$u_y = u_{y1} + zu_{y2} + z^2u_{y3} + z^3u_{y4} + (-1)^k \frac{2z_k}{h_k} u_{yZ}$$

$$u_z = u_{z1} + zu_{z2} + z^2u_{z3} + z^3u_{z4} + (-1)^k \frac{2z_k}{h_k} u_{zZ}$$



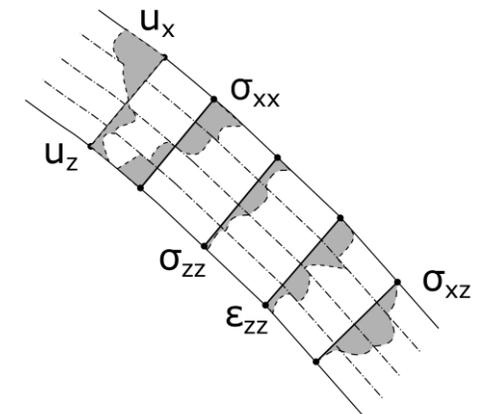
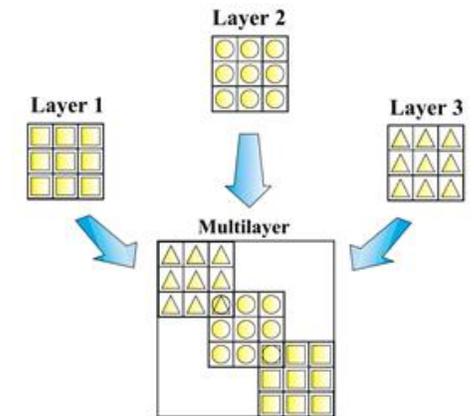
# Layer-Wise Approach (LW)

- LW leads to **zig-zag** distributions with the **additional** benefit of having **independent layers**
- **DOF** depend on the number of **layers**
- LW leads to **higher computational costs**, but its adoption is **necessary** in many cases, e.g., the precise detection of stress in each layer, **high-stress gradients** along the thickness, **free-edge**, and **local effects**
- Each **k-th layer** has its set of DOF
- Typical choices for  $F_\tau$  are **Lagrange or Legendre** polynomials
- For compatibility conditions at the interfaces, **'1' and '2'** variables are **displacements at the top and bottom** of the interface
- Usually, **third- or fourth-order** terms guarantee **quasi-3D accuracy**
- **Transverse stresses** are very **well detected** with **minor interlaminar discontinuities**

$$u_x^k = F_1 u_{x1}^k + F_2 u_{x2}^k + \sum_{\tau=3}^N F_\tau u_{x\tau}^k$$

$$u_y^k = F_1 u_{y1}^k + F_2 u_{y2}^k + \sum_{\tau=3}^N F_\tau u_{y\tau}^k$$

$$u_z^k = F_1 u_{z1}^k + F_2 u_{z2}^k + \sum_{\tau=3}^N F_\tau u_{z\tau}^k$$



Carrera E., Journal of Applied Mechanics (1998), 65, pp. 820-828

J.N. Reddy. Mechanics of laminated composite plates and shells. Theory and Analysis. CRC Press, 2004

# Models based on the Reissner Mixed Variational Theorem (RMVT)

- In mixed formulations, displacements and **stresses are primary variables**
- RMVT is one of the **most powerful tools** for composites, primary variables: the **displacements and the transverse stresses** [1]
- The use of RMVT is **independent of the type of expansion** adopted, Taylor expansions, zig-zag, layer-wise approaches, and **combination thereof can be used**. The example on the right is LW
- RMVT is **convenient** for multilayered structures as the **interlaminar continuity of transverse stresses** and **zig-zag displacements** are easily implementable
- The **post-processing** for transverse stresses is **not necessary**
- An **RMVT third-order zig-zag, ESL model**, can provide **3D-like accuracy in most cases** except for **severe local effects** in which the **layer-wise** version is preferable [2]

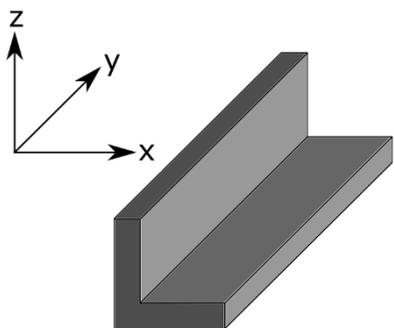
$$\begin{aligned}
 u_x^k &= F_1 u_{x1}^k + F_2 u_{x2}^k + \sum_{\tau=3}^N F_\tau u_{x\tau}^k \\
 u_y^k &= F_1 u_{y1}^k + F_2 u_{y2}^k + \sum_{\tau=3}^N F_\tau u_{y\tau}^k \\
 u_z^k &= F_1 u_{z1}^k + F_2 u_{z2}^k + \sum_{\tau=3}^N F_\tau u_{z\tau}^k \\
 \sigma_{xz}^k &= F_1 \sigma_{xz1}^k + F_2 \sigma_{xz2}^k + \sum_{\tau=3}^N F_\tau \sigma_{xz\tau}^k \\
 \sigma_{yz}^k &= F_1 \sigma_{yz1}^k + F_2 \sigma_{yz2}^k + \sum_{\tau=3}^N F_\tau \sigma_{yz\tau}^k \\
 \sigma_{zz}^k &= F_1 \sigma_{zz1}^k + F_2 \sigma_{zz2}^k + \sum_{\tau=3}^N F_\tau \sigma_{zz\tau}^k
 \end{aligned}$$

[1] E. Reissner, International Journal for Numerical Methods in Engineering, 23(2):193-198, 1986

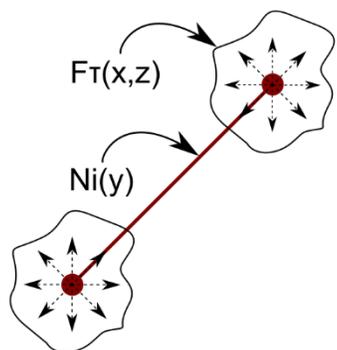
[2] E. Carrera, Archives of Computational Methods in Engineering, 10(3):216-296, 2003

# 1D and 2D

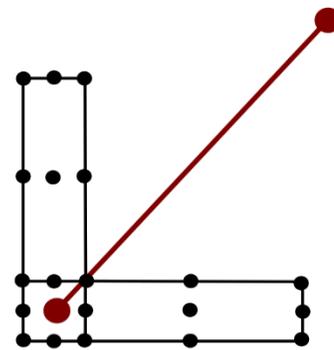
- So far, most of the equations focused on 2D – plate, shell- models. All theories can be used for **1D – beam – models**



3D Geometry

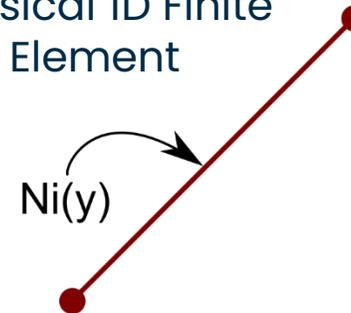


Taylor Expansion



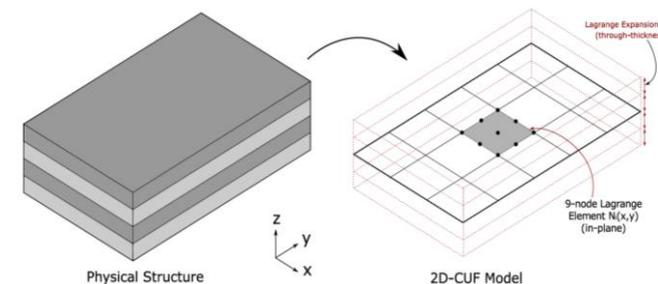
Lagrange Expansion

Classical 1D Finite Element



## Refined 1D – CUF models

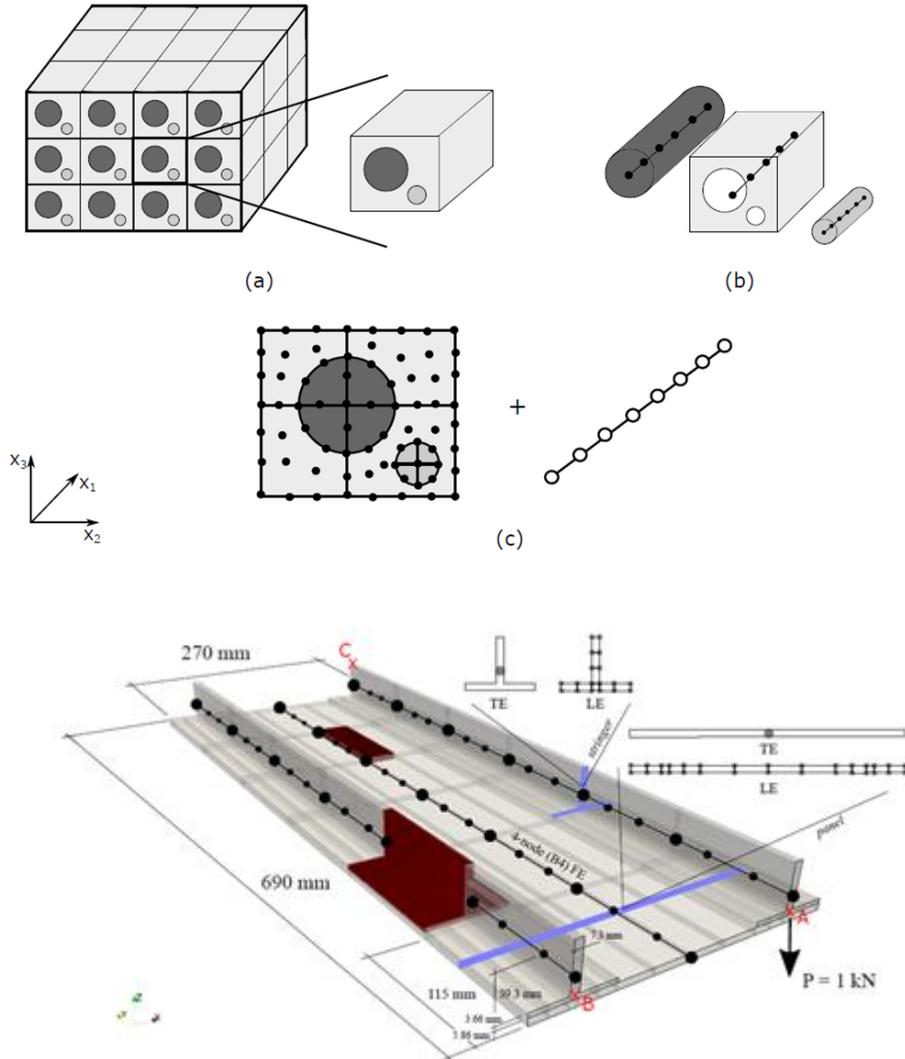
$$\begin{aligned}
 u_x &= u_{x1} + xu_{x2} + zu_{x3} + x^2u_{x4} + xzu_{x5} + \dots \\
 u_y &= u_{y1} + xu_{y2} + zu_{y3} + x^2u_{y4} + xzu_{y5} + \dots \\
 u_z &= u_{z1} + xu_{z2} + zu_{z3} + x^2u_{z4} + xzu_{z5} + \dots
 \end{aligned}$$



2D layer-wise modelling in CUF

# Component-Wise Approach (CW)

- All engineering structures have **multiple components** with various **scales, geometries and materials**
- The **concurrent, proper modeling** of components with different properties may be **difficult**
- Examples of critical **modeling issues: coupling of beam, plate, shell, and solid elements; accurate 3D geometrical modeling; homogenization of material properties;** etc.
- **CW** models use **1D CUF models for all components**, independently of their scale, geometry or material
- Advantages: **only pure displacements** as unknowns; **no need of reference surfaces** and **retaining of the 3D geometry;** **no need of homogenization techniques**



Carrera E. et al, AIAA Journal (2013), Vol. 51, 5, pp. 1255-1268

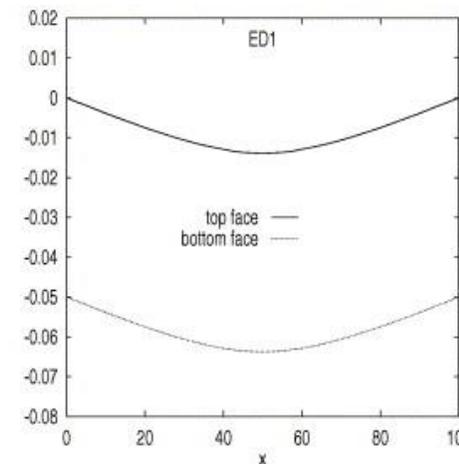
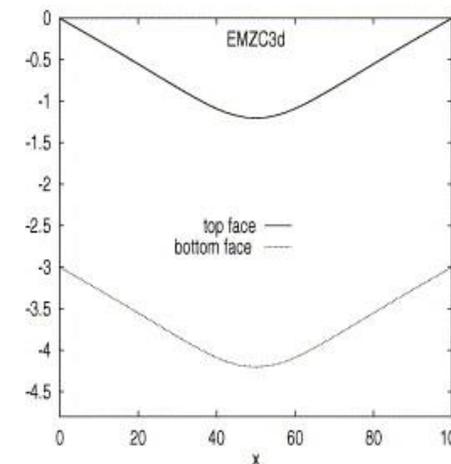
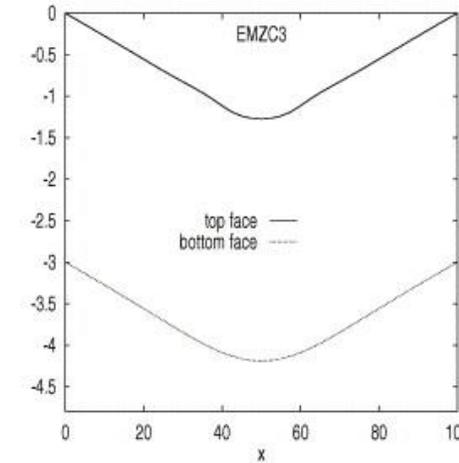
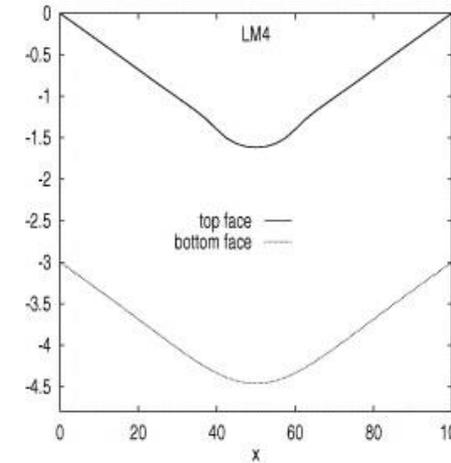
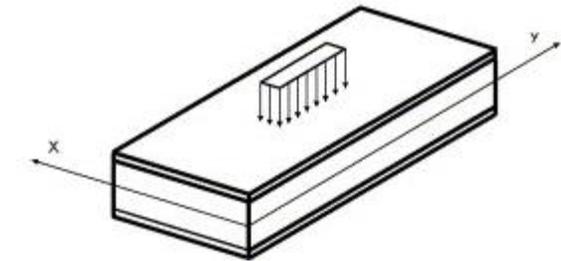
# PART I – Structural Theories

## NUMERICAL EXAMPLES

# Local Effects in Sandwich Panels

- Sandwich panels with  $a/h=10$  and **soft cores**: the **core deformability** can be, at least, one order of magnitude **higher than that of the faces**, and the **normal transverse deformation** is **necessary** to model the sandwich core
- Plots show the **transverse displacement in the loading region**, with:
  - LM4**: LW, fourth-order model based on RMVT
  - EMZC3**: ESL, third-order model based on RMVT with zig-zag and interlaminar continuity
  - EMZC3d**: EMZC3 without transverse normal strain
  - ED1**: ESL, first-order model

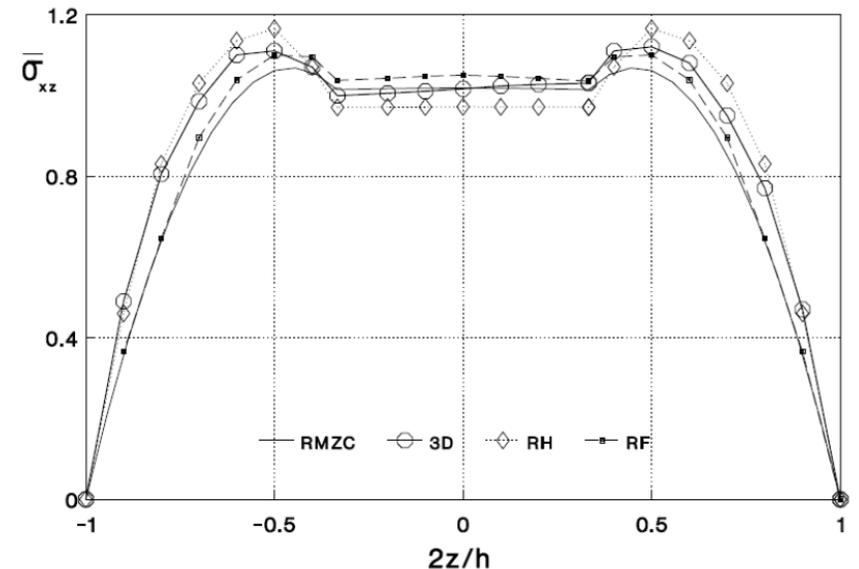
	Error wrt LM4	Top Face	Bottom Face
<b>LM4</b>	-	-	-
<b>EMZC3</b>	20%	18%	18%
<b>EMZC3d</b>	25%	18%	18%
<b>ED1</b>	99%	99%	99%



E. Carrera and A. Ciuffreda, Composite Structures, Volume 68, Issue 2, 2005, Pages 185-202

# Examples of $\sigma_{zz}$ effect

- Three-layer plate 0/90/0, simply-supported,  $a/h=4$ , distributed **load on the top surface**
- Plots show the **transverse shear stress** with:
  - **RMZC**: ESL, first-order model based on RMVT with zig-zag and interlaminar continuity
  - **3D**: exact solution
  - **RH and RF**: HOST without  $\sigma_{zz}$
- The **3D (exact) solution is not symmetric**
- The **inclusion of  $\sigma_{zz}$  is necessary** to capture this effect



Carrera E., International Journal for Numerical Methods in Engineering (1996), 39, pp. 1797-1820

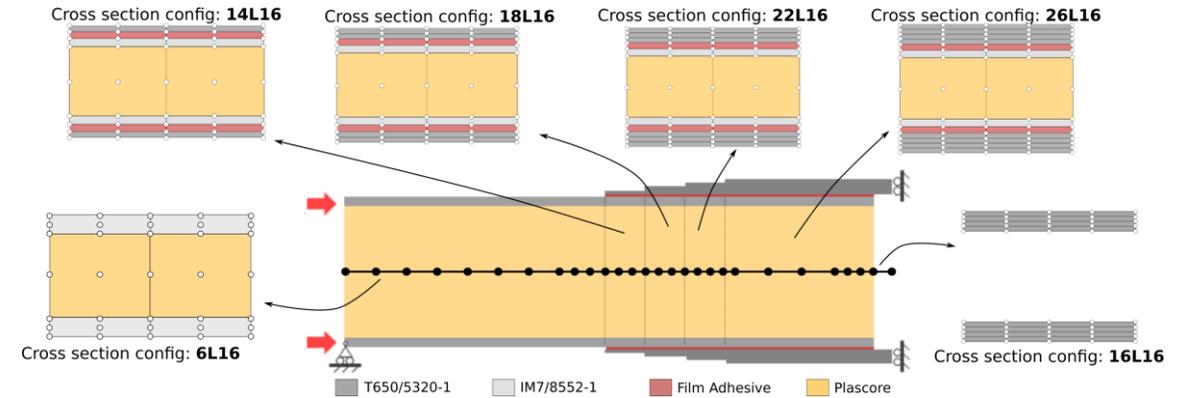
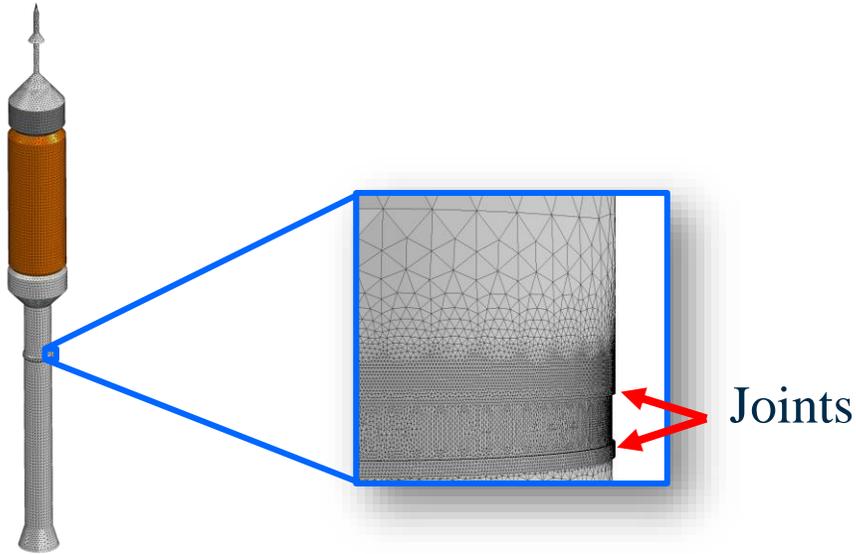
# Notes on Post-Processing of Transverse Stress

Transverse stress can be computed via **three approaches**

- 1) Stress components are evaluated using **the constitutive laws**
  - 2) The second method exploits the **indefinite equilibrium equations of 3D elasticity**. In the 2D case, the **in-plane stress components are integrated over the thickness** to obtain the transverse ones.
  - 3) Mixed formulations lead to **mixed cases** as assumed transverse stress is directly available - a priori - as primary variables or can be evaluated - a posteriori, as in the previous cases - via constitutive laws or by integration
- The use of **constitutive laws** has the advantage of **simplicity**, but **its accuracy is satisfactory in LW models** with, at least, **third-order expansions**.
  - The integration of the indefinite **equilibrium equations provides good accuracy**, but it may **sometimes fail to fulfill boundary conditions**. Furthermore, such a procedure **may be inaccurate as the number of layers increases**.
  - **LW mixed formulations** provide the **best results**

Carrera E., Composite Structures (2000), 48, pp. 245-260

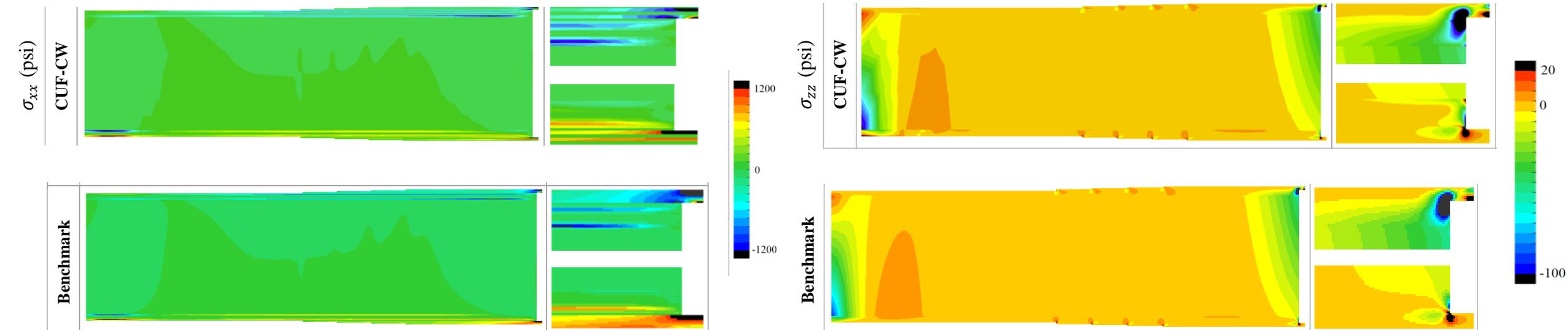
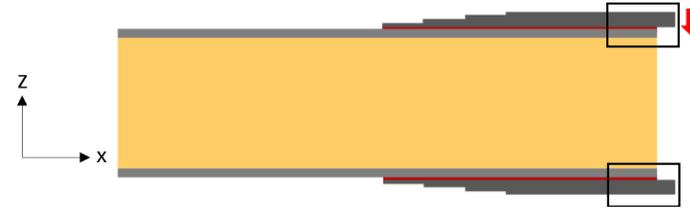
# Stress Analysis of Adhesively Bonded Joints 1/2



- CUF 1D CW: Mesh with **four-node beam elements** and Lagrange cross-section kinematics
- Converged 3D FEM as benchmark: **10 linear brick elements per layer and 20 for core**

Stapleton S.E. et al. (2021), MECHANICS OF ADVANCED MATERIALS AND STRUCTURES, Vol. 28, pp. 791-811

# Stress Analysis of Adhesively Bonded Joints 2/2

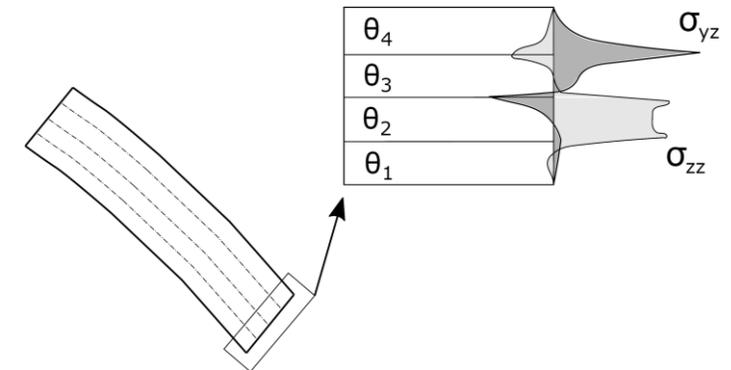


- **Accurate** capturing the **stress reversals** observed close to the **free surface**
- **Traction-free** conditions are **captured** along the free surface

Stapleton S.E. et al. (2021), MECHANICS OF ADVANCED MATERIALS AND STRUCTURES, Vol. 28, pp. 791-811

# Free-Edge Effects 1/3

- Defined as **stress states** at the **interfaces** between **dissimilar layers** in the vicinity of **geometrical or mechanical discontinuities** in the structure
- **Classical theories cannot** model free-edge effects; specific models continue to be developed for understanding the mechanics of this complex problem [1-3]
- The Poisson effect and the **various elastic moduli** of the material **over different plies** let them **behave differently in the in-plane direction**.
- To satisfy the **compatibility of the displacements** at the interfaces, **transverse stresses appear near the free edge**.
- A **correct evaluation** of these stresses is **necessary**, for instance, to prevent damage mechanisms such as **delamination**.
- Free-edge **3D stress fields** have **strong gradients** – as the **region is roughly equal to the total thickness of the laminate** – and may lead to remarkably **high peak values**.



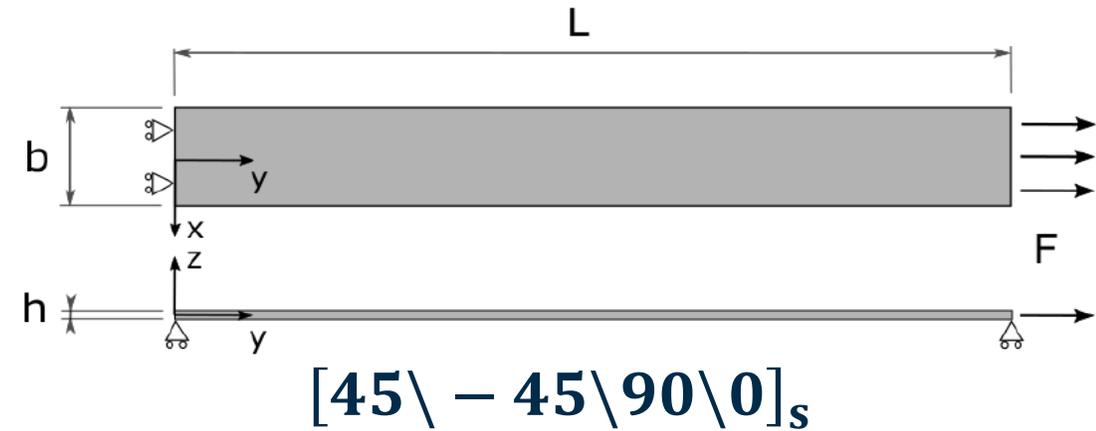
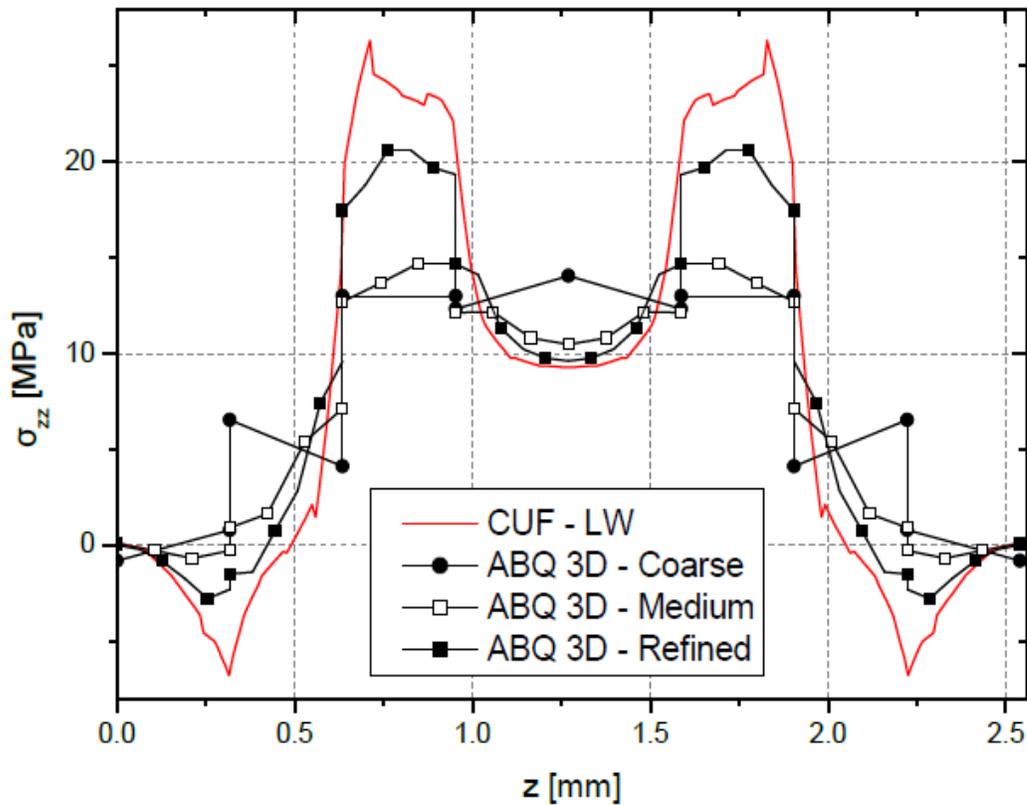
[1] T. Hayashi, vol 66, Transaction of Japan Society for Aeronautical Engineering and Space Science (1967), pp. 43-48

[2] A.H. Puppo, H.A. Evensen, J Compos Mater, 4 (2) (1970), pp. 204-220

[3] R.B. Pipes, N.J. Pagano, J Compos Mater, 4 (4) (1970), pp. 538-548

# Free-Edge Effects 2/3

- The peeling stress ( $\sigma_{zz}$ ) at the free-edge via **1D** is **more accurate than 3D** with five elements per node; **no stress recovery**; no aspect ratio constraints

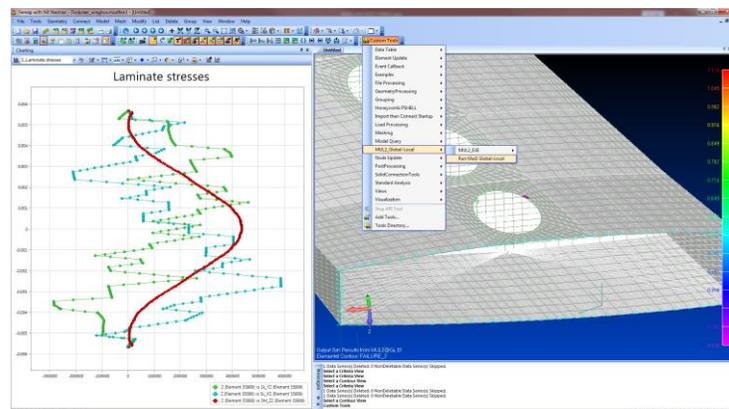


Model	DOF	Time (s)
CUF – LW (4 elements/ply)	<b>77,805</b>	<b>82</b>
3D FEM – Coarse (1 element/ply)	<b>168,237</b>	<b>27</b>
3D FEM – Medium (3 elements/ply)	<b>467,325</b>	<b>261</b>
3D FEM – Refined (5 elements/ply)	<b>3,501,933</b>	<b>3526</b>

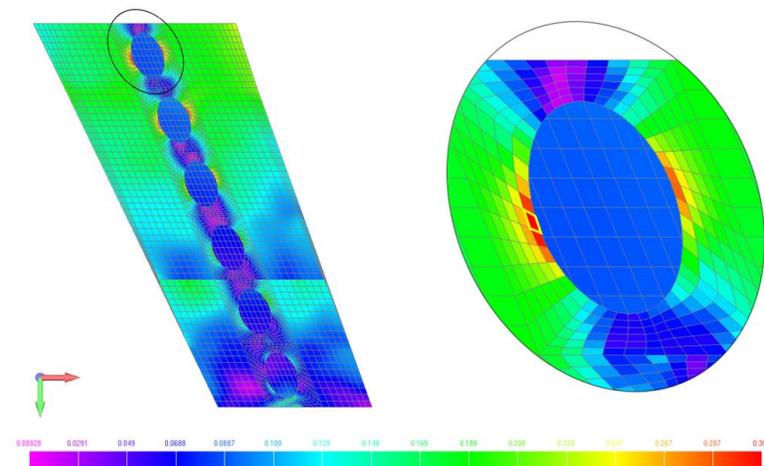
de Miguel A.G. et al. (2019), COMPOSITES. PART B, ENGINEERING, Vol. 168

# Free-Edge Effects 3/3

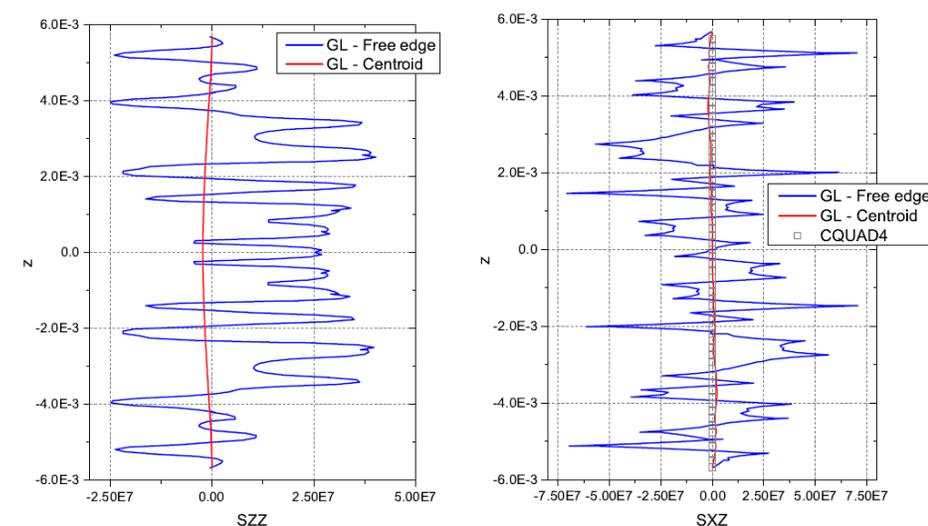
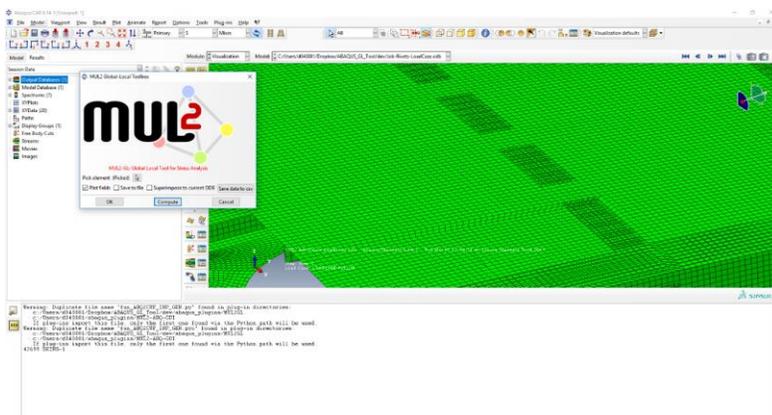
## Global-Local plug-in for NASTRAN and ABAQUS



Criteria	Shell Nastran Centroid	Mul2@GL Centroid	Mul2@GL Free Edge
Hoffman	0.307	0.307	0.510
LaRC5 Fiber Tension		0.208	0.206
LaRC5 Fiber Kinking		0.079	0.542
LaRC5 Matrix Failure		0.029	0.657



- **Global:** commercial **QUAD4**
- **Local:** **CUF LW**
- Centroid: center of the element on the edge
- **Dramatic increase of stress and failure indexes at the free-edge**



Carrera, E. et al. (2021), MECHANICS OF ADVANCED MATERIALS AND STRUCTURES, Vol. 28, pp. 1121-1127

Carrera, E. et al. (In Press), MECHANICS OF ADVANCED MATERIALS AND STRUCTURES, doi: [doi.org/10.1080/15376494.2019.1676938](https://doi.org/10.1080/15376494.2019.1676938)

# PART I Wrap-Up 1/2

- Feature: zig-zag displacements along the thickness

Guideline: The computationally **cheapest** option is the **ESL model with zig-zag functions**. Mixed models, both ESL and LW, provide such capabilities as well as 3D

- Feature: interlaminar continuity of transverse shear and normal stress

Guideline: **Mixed** ESL and LW **models** are the only models providing this capability. Otherwise, increasing the expansion order can improve the interface stresses, although the continuity is not guaranteed. Similarly, the use of **refined 3D meshes (5+ elements per layer)** can reach similar results

- Feature: Accurate description of transverse normal stress

Guideline: **LW models** or 3D are essential to obtain **accurate distributions of  $\sigma_{zz}$** . As stated above, only mixed models can guarantee the interface continuity of this stress component

- Feature: Accurate transverse stress without integration of equilibrium equations

Guideline: The use of **LW is fundamental** to obtain this capability. Mixed models provide the best accuracy but increasing the order of LW models without mixed features can improve the results significantly

# PART I Wrap-Up 2/2

- Minimization of the number of degrees of freedom

Guideline: **3D** is the most expensive option due to the **aspect ratio constraints** in which the **thickness of each ply** plays a decisive role. **2D or 1D LW models are less costly** but are still dependent on the number of layers. **ESL is the cheapest solution** as the number of variables is independent of the number of layers. As a **general guideline** based on a trade-off between accuracy and efficiency, **a third-order ESL mixed model with zig-zag functions** can be a **robust option** unless **severe local effects** are present as they **require LW capabilities**

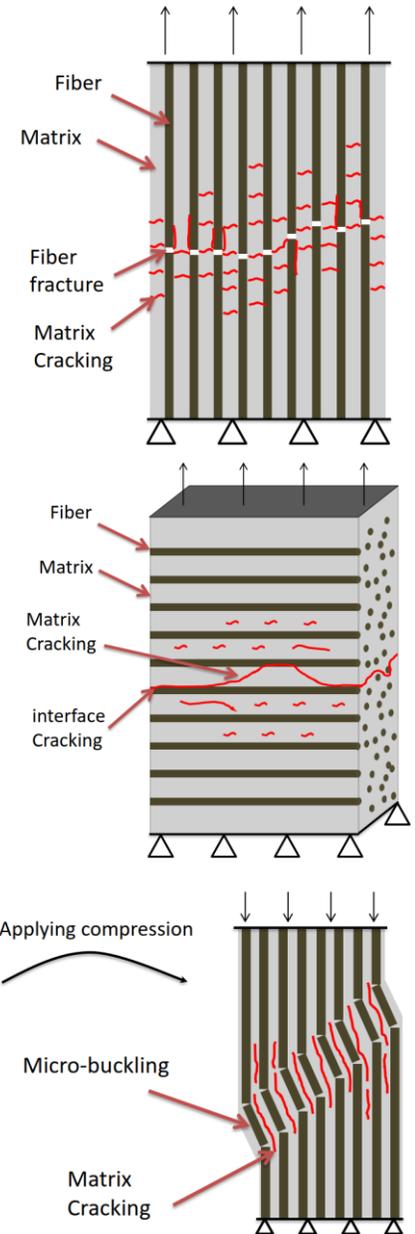
# PART II – Failure and Multiscale Analyses

## FAILURE ONSET AND PROGRESSION

# Failure Onset

- Failure mechanisms in composite structures are **complex phenomena** and **dependent** not only **on** the constituent **material properties**, but also **geometric features**, such as the **ply stacking sequence** and **fiber orientation**
- The **first step** in the analysis is the **determination of the failure onset**; that is, the **point-wise location** and the **load value**
- The typical strategy employs a **linear analysis** and **failure criteria** indicating the probability of **failure point-wise** and based on **strain and stress states**
- Failure criteria provide **failure indices**; values **greater than or equal to unity** indicate the **failure onset**
- One of the factors influencing the **accuracy of failure indexes** is the **quality of the strain or stress field** used in the analysis
- A **3D stress field** may be required to **accurately predict** the behavior of composite laminates

de Miguel A.G. et al. (2018) Composite Science and Technology, 167, pp 174–189; S.W. Tsai and H.T. Hahn, Winter annual Meeting of ASME, 1975, pp. 73–96; J.N. Reddy and A.K. Pandey, Comput. Struct. 25 (3) (1987) 371–393.; Y.S.N. Reddy and J.N. Reddy, Compos. Sci. Technol. 44 (3) (1992) 227–255; Z. Hashin, J. Appl. Mech. 47 (2) (1980) 329–334



# Failure Indexes

- Over the year, **many indexes have been proposed**, e.g., Puck , Hashin, LaRC03-05, etc.
- The **aim** of this webinar is not to discuss pro and cons of indexes, but **analyze the influence of structural modeling on the failure onset**
- The next examples has the **Hashin 3D** failure criteria [1] for the prediction of **ply failure** and the **mixed mode quadratic criteria** to determine the onset of **delamination** [2]

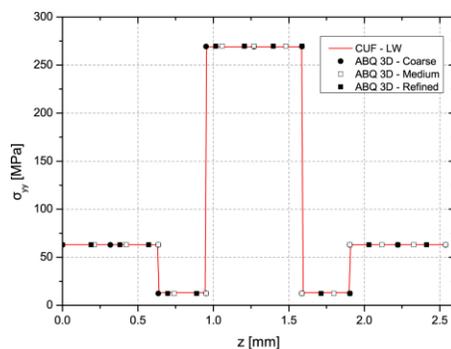
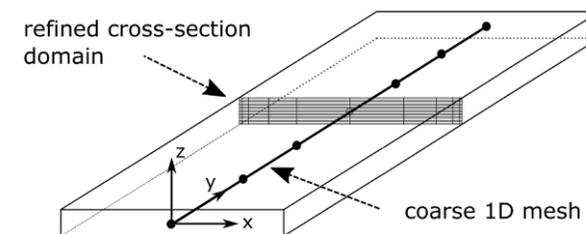
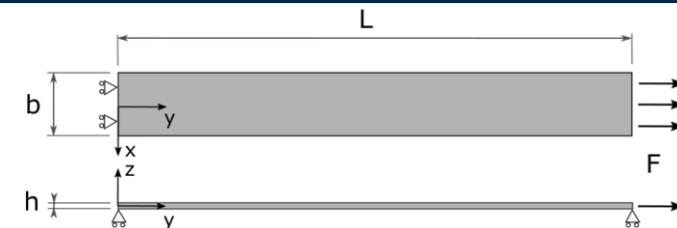
Fiber	Matrix	Delamination
T: $\left(\frac{\sigma_{11}}{X_T}\right)^2 + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} \geq 1$	T: $\frac{(\sigma_{22} + \sigma_{33})^2}{Y_T^2} + \frac{\sigma_{23}^2 - \sigma_{22}\sigma_{33}}{S_{23}^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} \geq 1$	$\left(\frac{\langle\sigma_{33}\rangle}{Z_T}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2 \geq 1$
C: $\left(\frac{\sigma_{11}}{X_C}\right)^2 \geq 1$	C: $\left[\left(\frac{Y_C}{2S_{23}}\right)^2 - 1\right] \left(\frac{\sigma_{22} + \sigma_{33}}{Y_C}\right) + \frac{(\sigma_{22} + \sigma_{33})^2}{4S_{23}^2} + \frac{\sigma_{23}^2 - \sigma_{22}\sigma_{33}}{S_{23}^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} \geq 1$	

T: Tension; C: Compression; X, Y: Material Strengths along the fiber and transverse directions; S: Shear material strengths; Z: Interlaminar normal strength

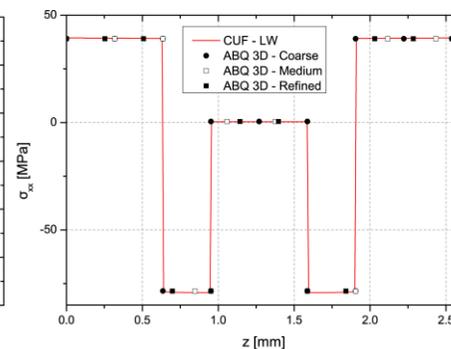
[1] Z. Hashin, J. Appl. Mech. 47 (2) (1980) 329–334; [2] J.C. Brewer, P.A. Lagace, J. Compos. Mater. 22 (12) (1988) 1141–1155

# Virtual Tensile Test 1/4

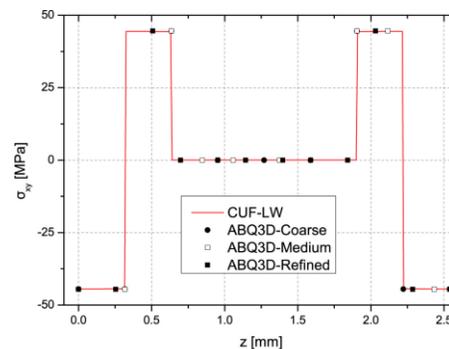
- Lamination:  $[45 \setminus -45 \setminus 90 \setminus 0]_s$
- **Failure indexes** from various models are compared
- **In-plane stress** components **coincide**
- **Differences** due to **transverse stress**



(a)  $\sigma_{yy}$



(b)  $\sigma_{zz}$



(c)  $\sigma_{xy}$

Model	DOF	Time (s)
CUF – LW (4 elements/ply)	<b>77,805</b>	<b>82</b>
3D FEM – Coarse (1 element/ply)	<b>168,237</b>	<b>27</b>
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3D FEM – Refined (5 elements/ply)	<b>3,501,933</b>	<b>3526</b>

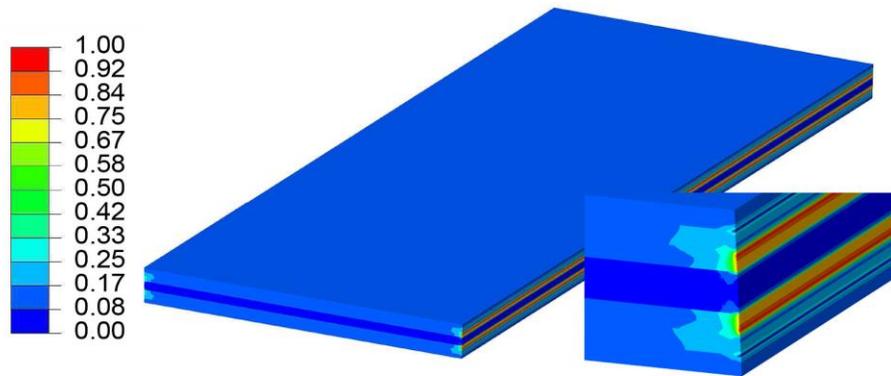
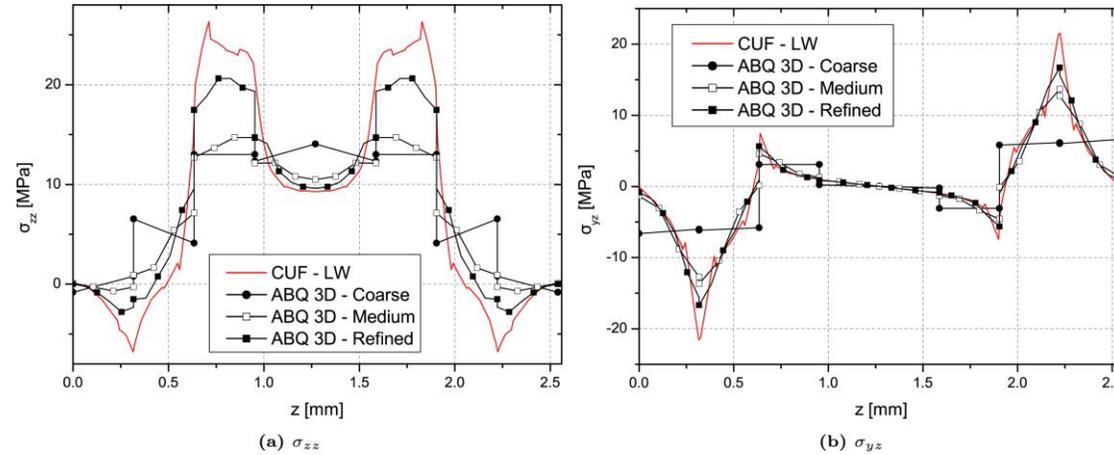
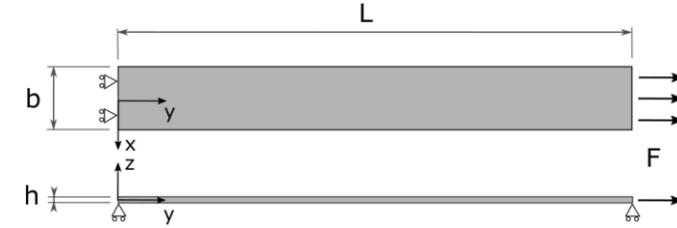
Model	Delamination	Matrix-T	Fiber-T	Fiber-T*
<b>Load [N]</b>	14288	11938	27178	34417
<b>CUF-LW</b>	1.00	1.00	1.00	1.00
<b>ABAQUS – Coarse</b>	0.28	0.40	0.70	0.94
<b>ABAQUS – Medium</b>	0.31	0.45	0.71	1.05
<b>ABAQUS – Refined</b>	0.60	0.73	0.82	1.07

\*Modified Hashin 3D criteria with only in-plane components

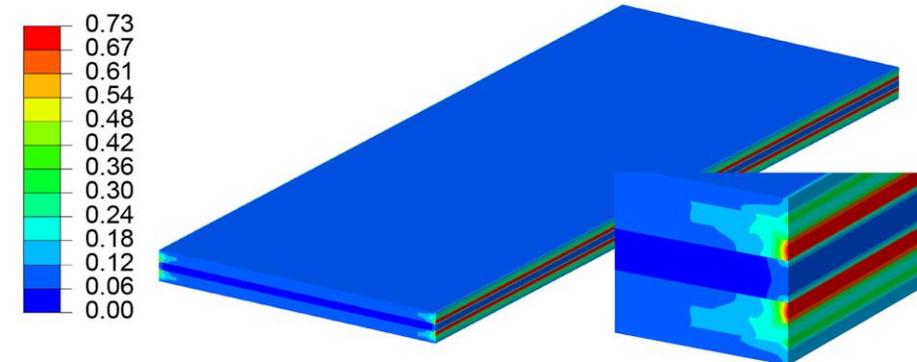
de Miguel A.G. et al. (2018) Composite Science and Technology, 167, pp 174-189

# Virtual Tensile Test 2/4

Failure onset along free-edges



(a) CUF-LW

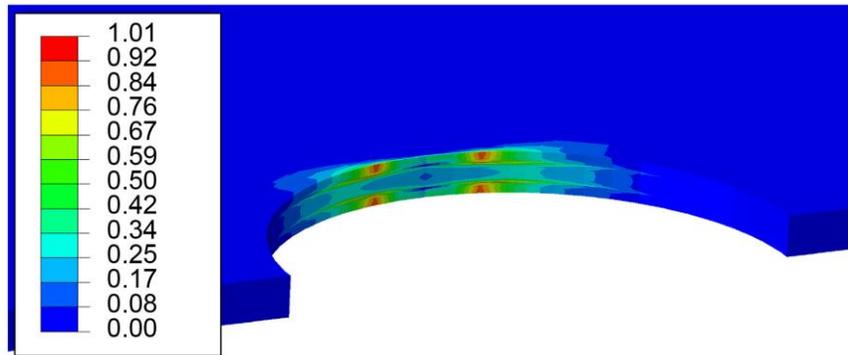
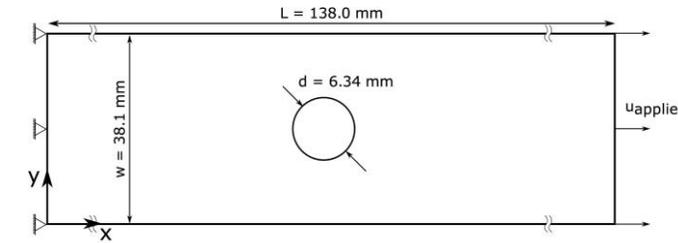


(b) ABQ3D - refined

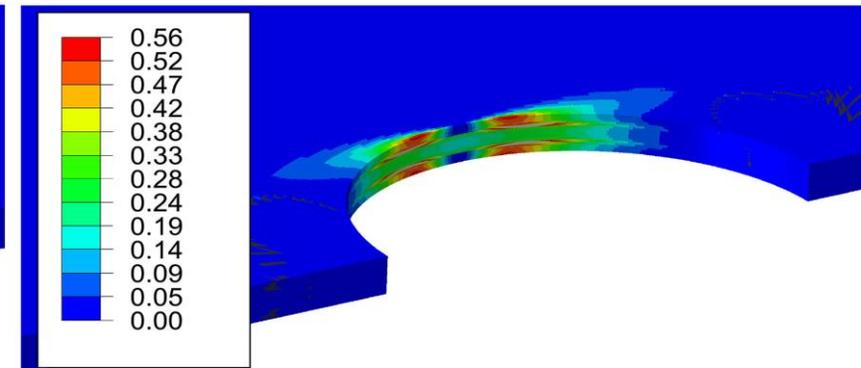
de Miguel A.G. et al. (2018) Composite Science and Technology, 167, pp 174-189

# Virtual Tensile Test 3/4

- Lamination:  $[45 \setminus -45 \setminus 90 \setminus 0]_s$
- CUF: 53346 DOF, ABAQUS (4 linear solid per layer): 1306977
- Matrix Tension FI:



(a) CUF-LW2 (53,346 DOF)

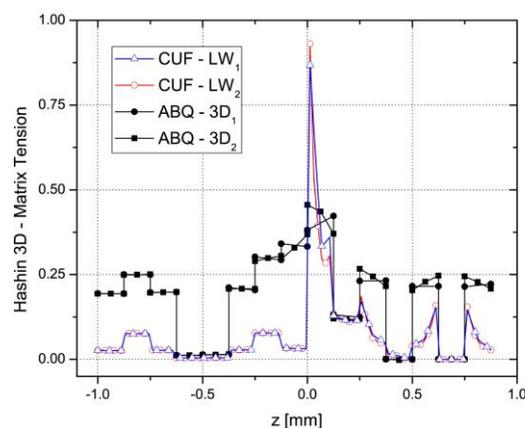
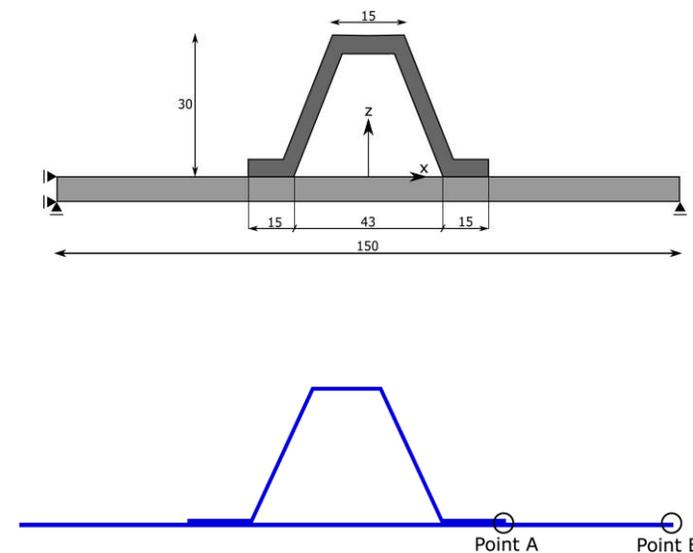


(b) ABQ3D-4L (1,306,977 DOF)

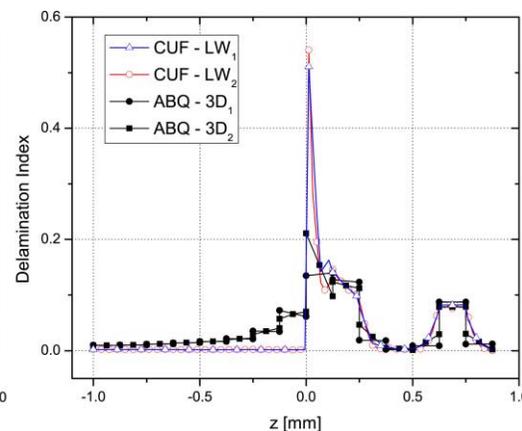
de Miguel A.G. et al. (2018) Composite Science and Technology, 167, pp 174-189

# Virtual Tensile Test 4/4

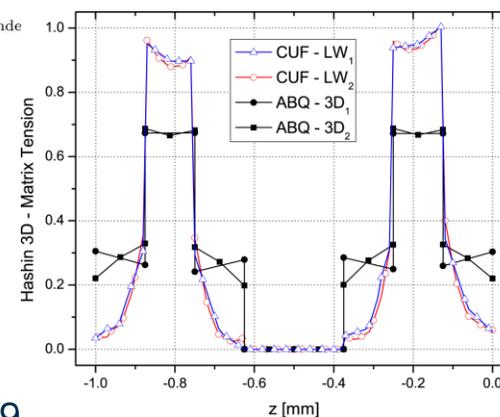
- Laminations: skin:  $[45\backslash 90\backslash -45\backslash 0]_s$ ; stringer:  $[-45\backslash 0\backslash 45\backslash 0\backslash 45\backslash 0\backslash -45]$
- Refined models: **CUF: 277326 DOF, ABAQUS (2 linear solid per layer): 4560150**



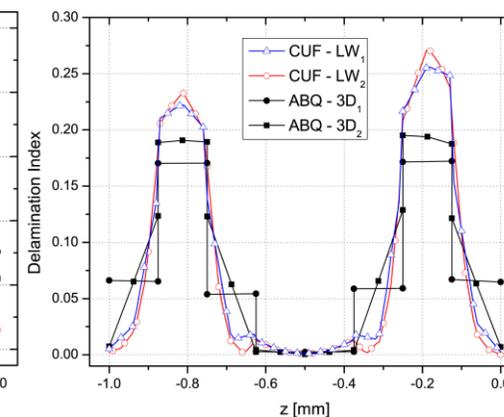
(a) Hashin 3D - Matrix Tension



(b) Delamination Index



(a) Hashin 3D - Matrix Tension



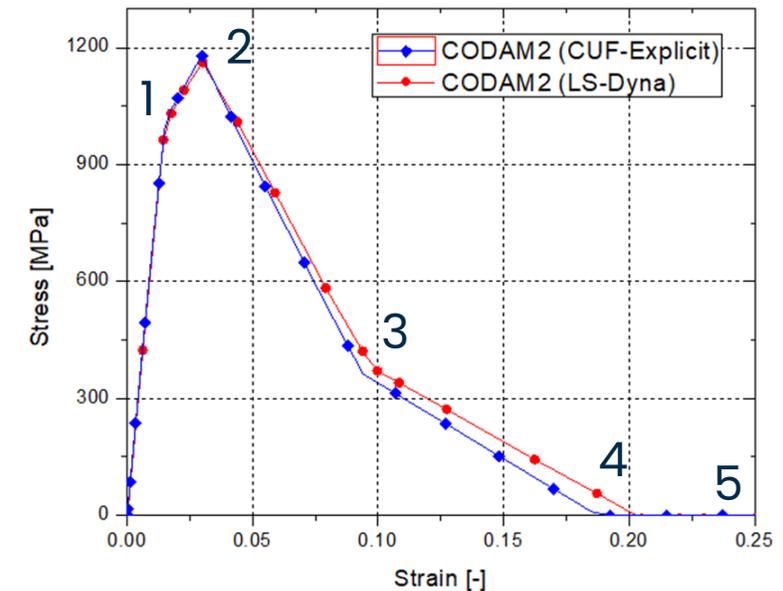
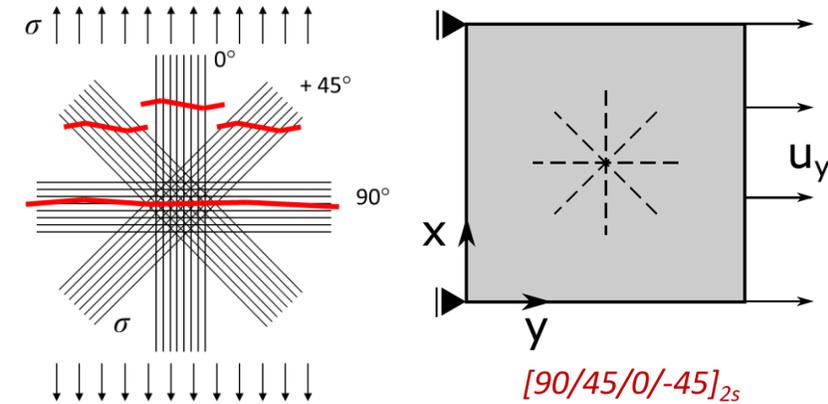
(b) Delamination Index

Model	Delamination	Matrix-T	Fiber-T
<b>Free-Edge Stringer</b>			
CUF-LW	0.54	0.93	0.19
ABAQUS	0.21	0.44	0.18
<b>Free-Edge Skin</b>			
CUF-LW	0.24	1.01	0.76
ABAQUS	0.19	0.68	0.21

de Miguel A.G. et al. (2018) Composite Science and Technology, 167, pp 174-189

# Damage Progression

- After the onset of failure, **material properties are degraded** and there is the **progressive extension** of the **damaged area**
- Basic example: **intralaminar damage under tension**
  1. Matrix damage initiation in the 90° layer
  2. Fiber damage initiation in the 0° layer
  3. The fibers of the ±45° layers start to degrade
  4. The fiber damage in the 0° layer saturates
  5. Damage is completely saturated, no more load-carrying capacity
- As the loading, e.g., **compression**, or the **stacking sequence changes**, the progression of damage can change. Also, this example does not include **delamination**
- The area under the  $\sigma - \epsilon$  curve is the **fracture energy release rate density**,  $g_f$



Forghani, A. et al. (2013), Journal of Composite Materials, 47(20-21), 2553-2573

Nagaraj, M.H. et al. (2020), COMPOSITES. PART B, ENGINEERING, Vol. 190

# Damage Progression: Modeling Strategies

Approaches:

- **Discrete** damage modelling (xFEM, phase field, floating node etc.)
- **Continuum** damage modelling – smeared approach
- **Combined** approach – Continuum damage (intralaminar) + discrete damage (interlaminar)

Challenges:

- **Computational cost** – analysis time and memory required
- Trade-off between cost and **fidelity**
- **Scalability** – aspect ratio constraints of finite elements

Bouvet, C. et al. (2009), International Journal of Solids and Structures, 46(14-15), 2809-2821.

Kim, E. H. et al. (2013), Composite Structures, 95, 123-134.

Lopes, C. S. et al. (2009), Composites Science and Technology, 69(7-8), 937-947.

# Intralaminar Continuum Damage

- COmposite DAMage model (**CODAM2**) developed at the University of British Columbia [1]
- Stress-based failure initiation criteria (**Hashin**)
- **Crack-band** approach [2]: **smearing** the gradual effect of **micro-cracking** over a band of material with known dimension. The **scaling of fracture energy** is necessary to **avoid mesh-dependency and ensure total energy release** upon complete failure equal to **fracture toughness**
- **Damage variables** used to **reduce the stiffness matrix**
  1. The damage variable is a function of the strain state,  $d = f(\epsilon)$
  2. The material properties are degraded according to the damage variable,  $C = f(d)$
  3. A nonlinear analysis is required
- Solved using an **explicit time integration scheme**
- Available in **LS-DYNA** and in **MUL2**

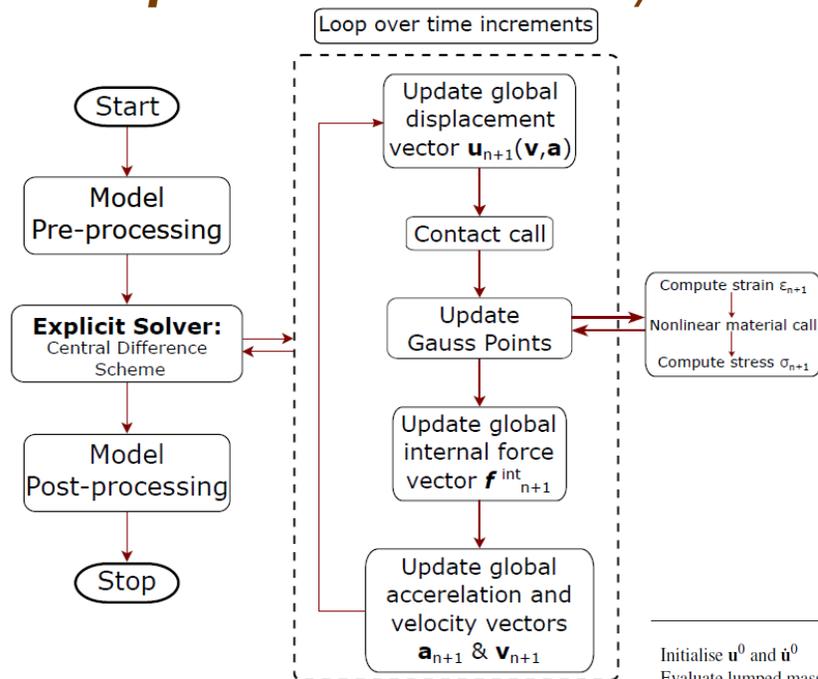
$$C^{dam} = \frac{1}{\Delta} \begin{bmatrix} (1 - R_2 v_{23} v_{32}) R_1 E_1 & (v_{21} + v_{23} v_{31}) R_1 R_2 E_1 & (v_{31} + R_2 v_{21} v_{32}) R_1 E_1 & 0 & 0 & 0 \\ & (1 - R_1 v_{31} v_{13}) R_2 E_2 & (v_{32} + R_1 v_{31} v_{12}) R_2 E_2 & 0 & 0 & 0 \\ & & (1 - R_1 R_2 v_{21} v_{12}) E_3 & 0 & 0 & 0 \\ & & & \Delta R_1 R_2 G_{12} & 0 & 0 \\ & & & & \Delta G_{23} & 0 \\ & & & & & \Delta G_{13} \end{bmatrix}$$

*sym.*

[1] Forghani, A. et al. (2013), Journal of Composite Materials, 47(20-21), 2553-2573; [2] Bazant, Z. and Oh, B. H. (1983), Materials and Structures, 16:155-177;

# Implicit vs Explicit

## Explicit nonlinear analysis in CUF

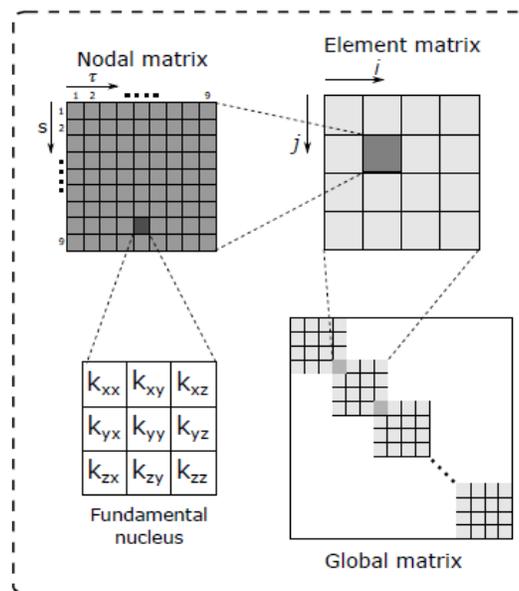


### Central difference scheme algorithm

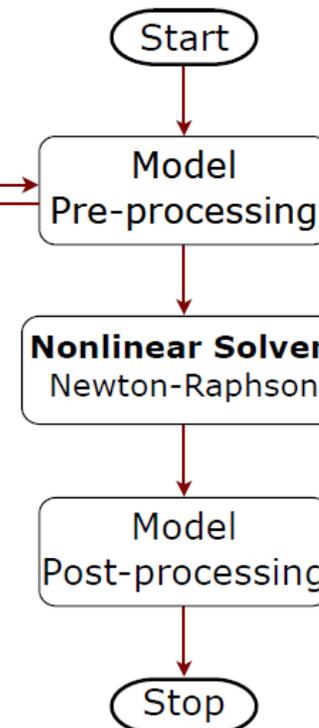
Initialise  $\mathbf{u}^0$  and  $\dot{\mathbf{u}}^0$   
 Evaluate lumped mass matrix  $\mathbf{M}$   
 Compute initial mid-interval velocity:  $\dot{\mathbf{u}}^{\frac{1}{2}\Delta t} = \dot{\mathbf{u}}^0 + \frac{1}{2}\Delta t \ddot{\mathbf{u}}^0$

For each time increment:

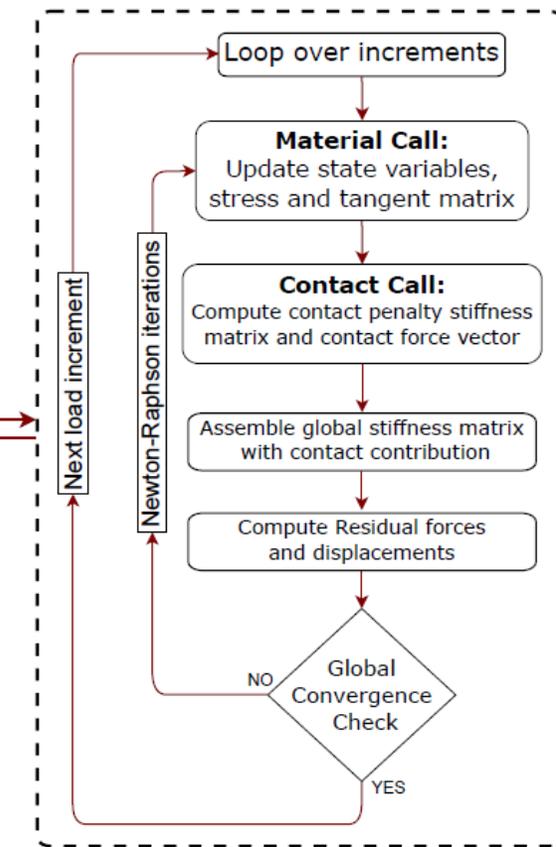
1. Evaluate new displacements:  $\mathbf{u}^{t+\Delta t} = \mathbf{u}^t + \Delta t \dot{\mathbf{u}}^{t+\frac{1}{2}\Delta t}$
2. Compute displacement increment:  $\Delta \mathbf{u} = \mathbf{u}^{t+\Delta t} - \mathbf{u}^t$
3. At each integration point:  
 → Compute updated strains:  $\boldsymbol{\epsilon}^{t+1} = \boldsymbol{\epsilon}^t + \mathbf{B}\Delta \mathbf{u}$   
 → Compute updated stress:  $\boldsymbol{\sigma}^{t+1} = \mathbf{C}^{sec}\boldsymbol{\epsilon}^{t+1}$
4. Compute internal force vector:  $\mathbf{F}_{int}^{t+\Delta t} = \int_V \mathbf{B}^T \boldsymbol{\sigma}^{t+\Delta t} dV$
5. Compute new accelerations:  $\ddot{\mathbf{u}}^{t+\Delta t} = \mathbf{M}^{-1} \{ \mathbf{F}_{ext}^{t+\Delta t} - \mathbf{F}_{int}^{t+\Delta t} \}$
6. Compute new mid-interval velocities:  $\dot{\mathbf{u}}^{t+\frac{3}{2}\Delta t} = \dot{\mathbf{u}}^{t+\frac{1}{2}\Delta t} + \Delta t \ddot{\mathbf{u}}^{t+\Delta t}$



### Stiffness matrix assembly in CUF



## Implicit nonlinear analysis in CUF



# Interlaminar Damage: Delamination

- Delamination is one of the **most predominant failure** in laminated composite structure
- Typical causes: run-way debris **impact**, tool-drop during maintenance, **high interlaminar stresses** and geometric discontinuities, e.g., **free-edge**
- Modeling strategies: **cohesive** zone models [1, 2], **virtual crack closure** technique [3]
- Precursor to **precise delamination** analysis: **accurate transverse fields**
- Challenges:
  1. Accurate stress resolution dictates the need of **computationally intensive FE models**
  2. Appropriate **constitutive modeling** of the cohesive surface
  3. Extremely **refined mesh** near the cohesive zone
  4. Convergence issues in **tracing the equilibrium path**
  5. Scalability to **large-scale structures** is still computationally **infeasible**

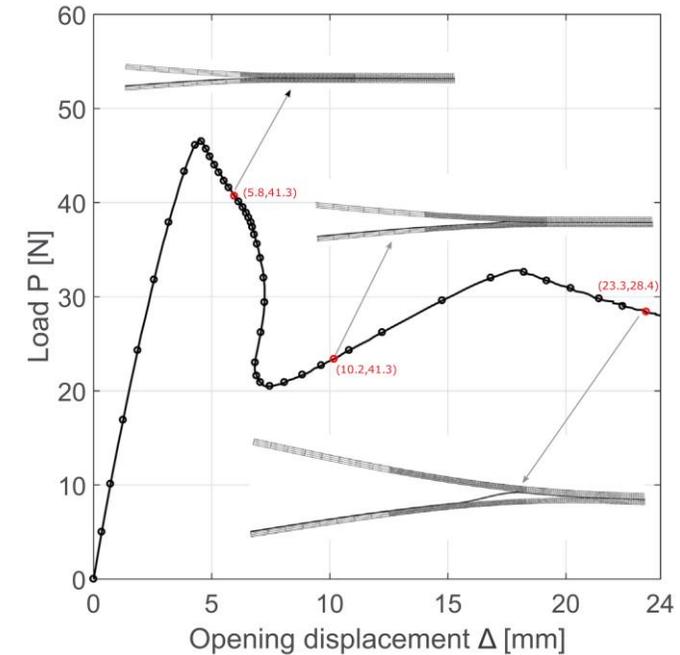
[1] Dugdale, D. (1960), Journal of the Mechanics and Physics of Solids, 8(2):100 – 104

[2] Barenblatt, G. I. (1962). Volume 7 of Advances in Applied Mechanics, pages 55 – 129

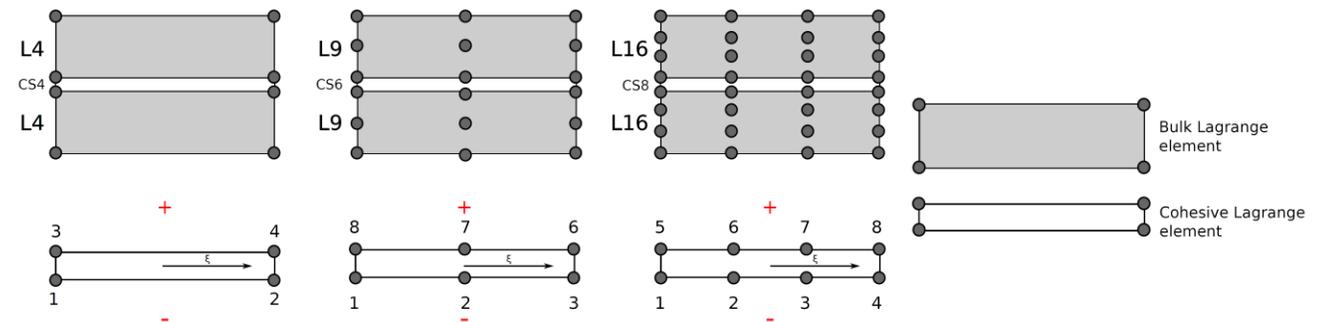
[3] Rybicki, E. and Kanninen, M. (1977), Engineering Fracture Mechanics, 9(4):931 – 938.

# Cohesive Elements

- Basic idea: existence of a **narrow band of vanishing thickness** ahead of a **crack tip**
- The narrow band has **two virtual surfaces across the crack tip** with presence of **resistive cohesive forces**
- Cohesive **constitutive law** relates the **cohesive traction** to the **displacement jump across the interface surface**
- The area under the **traction-separation curve** corresponds to **fracture toughness**
- Failure **onset**:  $\left(\frac{\langle\sigma_{33}\rangle}{Z_T}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2 \geq 1$

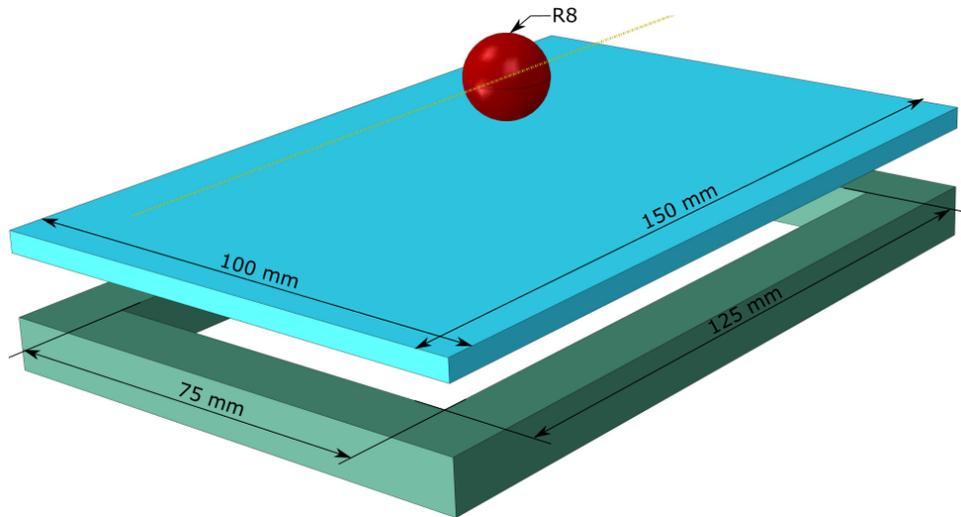


Kaleel I. et al. (2020), Composite Structures, Vol. 235



# Example: Low-Velocity Impact 1/2

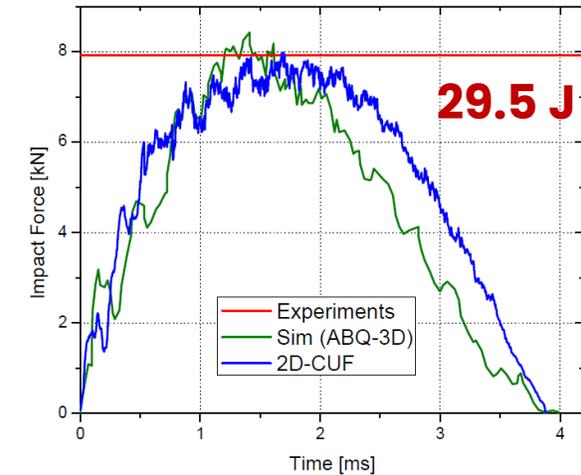
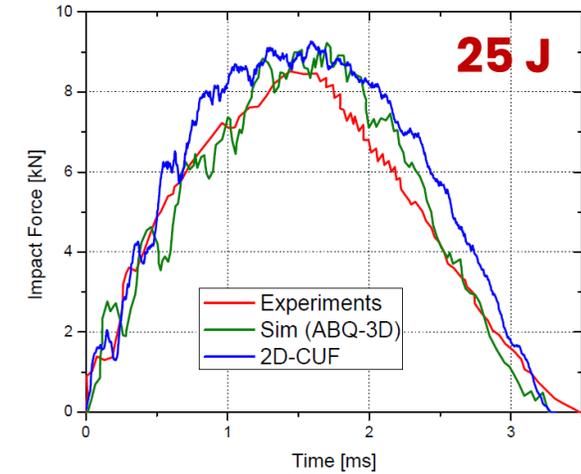
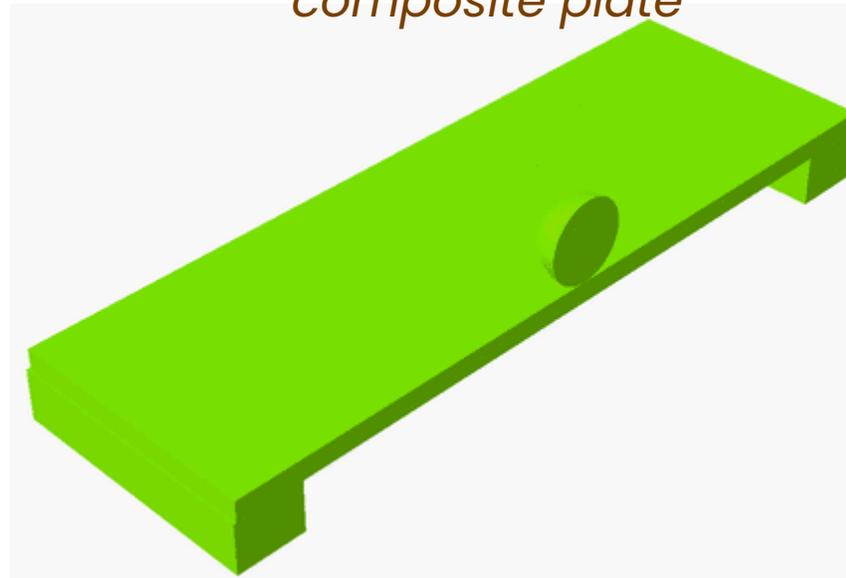
- In a low-velocity impact event, **all damage mechanisms** can be activated
- Besides the modeling challenges seen above, to model impact the implementation of **contact** algorithms are necessary



$[0_2/45_2/90_2/-45_2]_2$  T laminate = 4.16 mm  
 T700/M21 CFRP T ply = 0.26 mm

**Quasi-isotropic laminate**

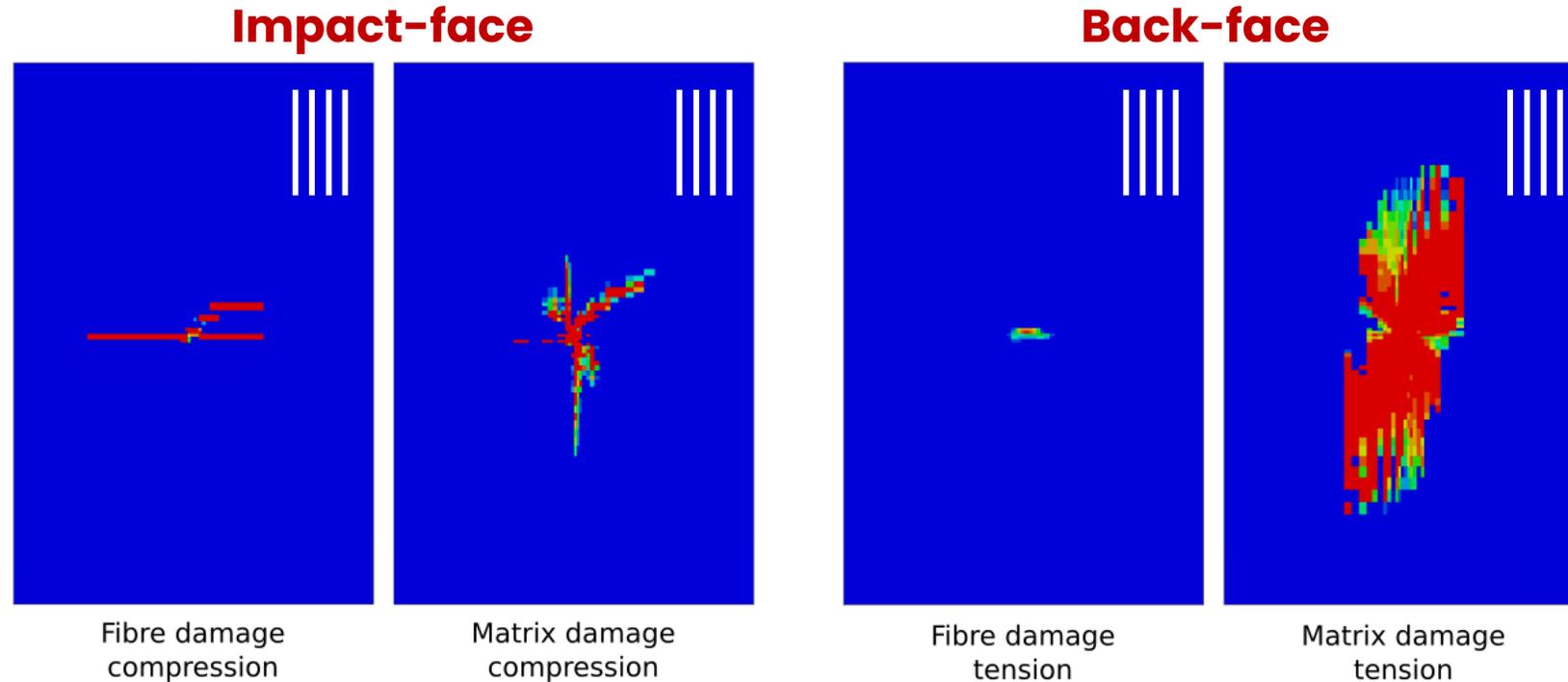
*Vertical deflection  $u_z$  of composite plate*



*Impact force – time response*

Nagaraj, M. H. et al. (2020), INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS, Vol. 127

# Example: Low-Velocity Impact 2/2



Damage profile of  $[0_2/45_2/90_2/-45_2]_2$  quasi-isotropic laminate under 29.5J impact

## COMPUTATIONAL COST

### Ref ABQ-3D:

~10 hours @ 32 CPUs

### 2D-CUF (LW):

~4 hours @ 4 CPUs

Nagaraj, M. H. et al. (2020), INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS, Vol. 127

# PART II – Failure and Multiscale Analyses

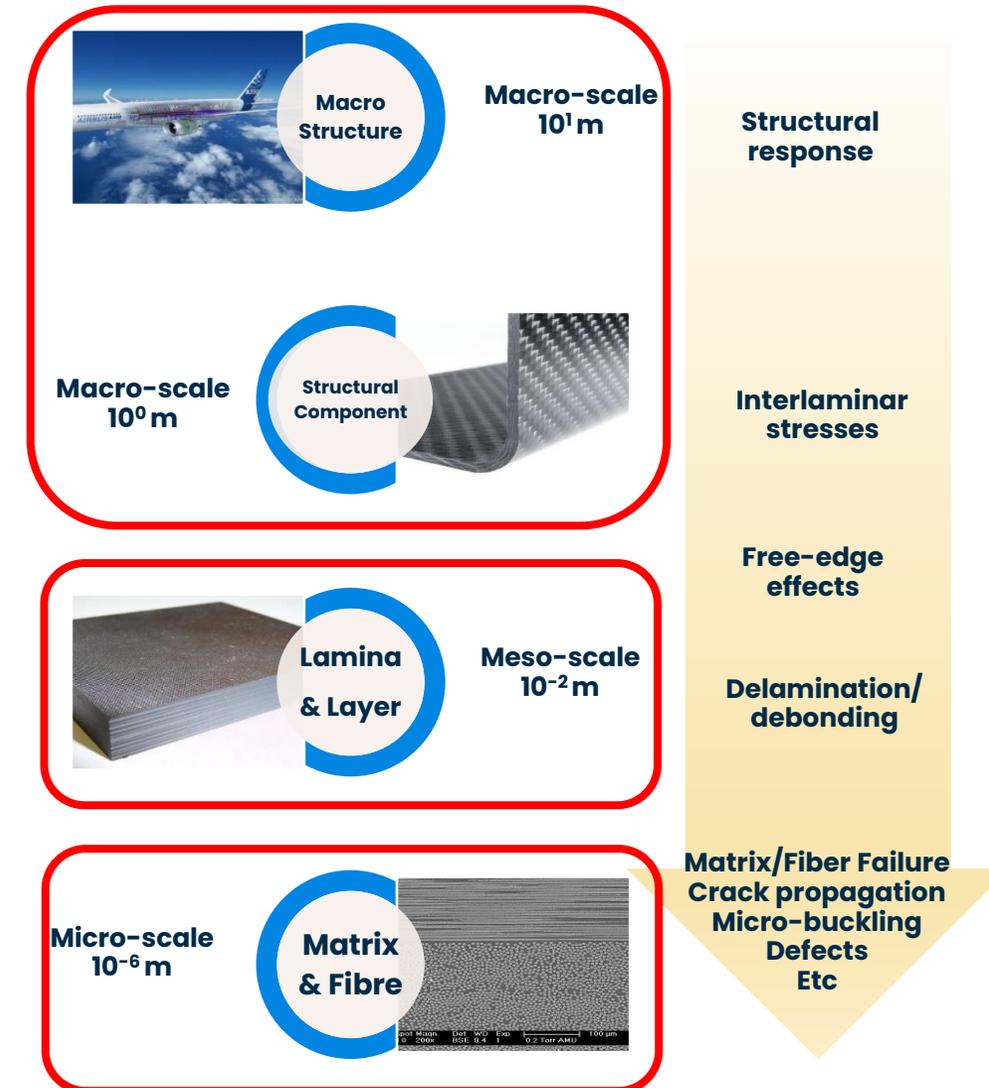
## MICROMECHANICS

# Motivations

- The **homogeneous continuum** is assumed to **retain** its bulk material **properties irrespective of the scale**.
- In case of real materials, the **assumption starts to fade at lower scales** as they turn **heterogeneous in nature**
- Hierarchical material systems, such as composites, necessitate micromechanics to **capture lower scales phenomena** for reliable simulations
- Micromechanics captures the **effective behavior of heterogeneous systems** through **homogenization** (up-scaling) and **localization** (down-scaling) for a given representative volume element (**RVE**)
- Micromechanics often **enriches physically nonlinear simulations** such as progressive failure in composites, as they are greatly stimulated by **underlying lower scale features**
- Effectiveness is greatly influenced by **proper constitutive modeling** at lower scales and the ability of these models to generate **accurate local fields** at the constituent level
- The former requires rigorous validation through **in-situ experiments**
- The latter greatly depends on the **mathematical approaches employed**

Bednarczyk, B. A. et al. (2010), AIAA Journal, 48(7):1367–1378

Liu, X. et al. (2018), Vision 2040: A roadmap for integrated, multiscale modeling and simulation of materials and systems. NASA Glenn Research Center



# Aims and Approaches

- ISSUE: the **direct numerical simulation** of a composite structure accounting for **all scales** concurrently **is not feasible**

## AIMS

- RVE: a **volume of the structure** including a representative description of the microscale **architecture of the constituents**, e.g., **fibers, matrix, honeycomb packing, voids, defects**
- Homogenization (up-scaling): provide homogenized material **properties for the upper scales** starting **from the constituent properties**
- Localization or de-homogenization (down-scaling): provide **stress and strain fields at the microscale** for a given **macroscale load**
- The micromechanical formulations intends to **capture the explicit heterogeneous nature of the microstructure** and provide an **effective continuum to the higher-scale** by defining **representative volume elements (RVE)**

## STRATEGIES AND CHALLENGES

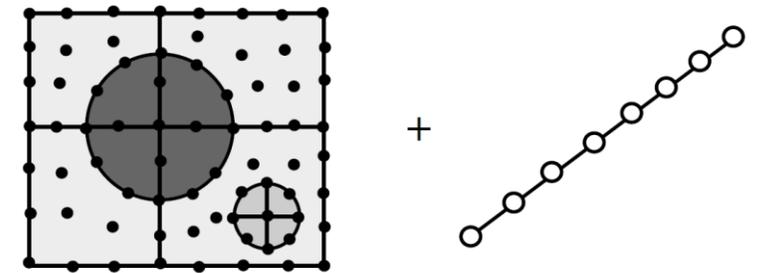
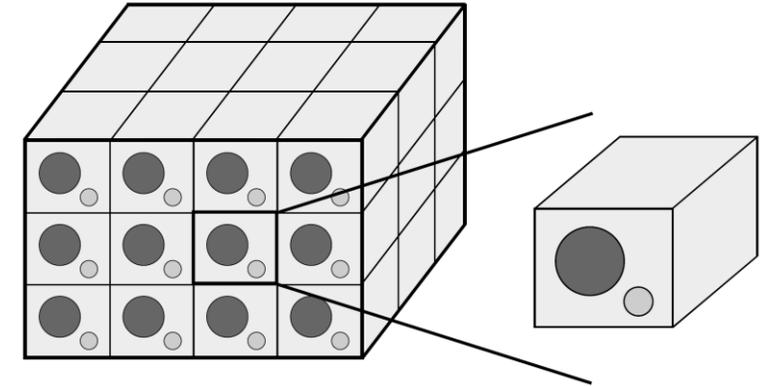
- **Analytical, semi-analytical** and full numerical methods are available. In MUL2, **fully numerical methods** via FE
- The main **challenge** is to develop a model having **accurate homogenization and de-homogenization** while **keeping the computational cost low** to favor **multiscale and nonlinear analyses**
- The main **novelty in MUL2** is the use of **1D models instead of 3D** leading to **superior de-homogenization and computational cost performances**

Aboudi J., Arnold S., Bednarczyk B. (2021), Practical Micromechanics of Composite Materials, Butterworth-Heinemann

W. Yu, J Mech Mater Struct, 11 (4) (2016), pp. 379-411

# RVE's Basic Concepts

- Define the **RVE architecture**; typical choices: one fiber + matrix; multiple fibers + matrix; honeycomb; textiles
- Build an **RVE model**; typical choices: **3D FEM** or, with CUF, **1D Component-Wise** models
- Although in composites the actual **fiber distribution is quite random** across the cross-section, the micromechanical formulation is postulated based on the **assumption of periodicity**. An RVE is the basic unit of the periodic structure. To maintain compatibility of displacements along different faces of RVE, **periodic boundary conditions** are applied
- In a composite, fibers and matrix have **vastly different properties**. All the **RVE are identical** and must have **identical stress and strain fields**; that is, the stress and strain fields are **periodic**
- The periodicity boundary conditions are imposed to displacements of **external faces**



Sun, C. T. and Vaidya, R. S. (1996), Composites Science and Technology, 56(2):171–179

Xia, Z. et al. (2003), International Journal of Solids and Structures, 40(8):1907–1921

# Homogenization and Localization

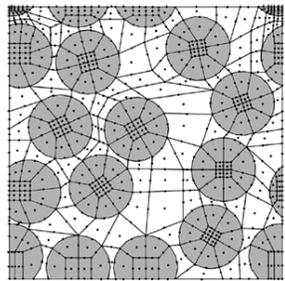
- Homogenization provides **effective properties** based on the **individual constituents** of the RVE; homogenization is achieved through **volume averaging** the **stress and strain** fields:  $\bar{\epsilon}_{ij} = \frac{1}{V} \int \epsilon_{ij} dV$ ,  $\bar{\sigma}_{ij} = \frac{1}{V} \int \sigma_{ij} dV$ ,  $V$  is the volume of the RVE,  $\sigma_{ij}$  and  $\epsilon_{ij}$  are the **local stress** and **local strain** within the individual constituents of the RVE, respectively
- Six individual **unit strains are applied** and the **effective material matrix**,  $\bar{C}$ , of the homogenized RVE is populated through  $\bar{\sigma} = \bar{C}\bar{\epsilon}$
- Localization requires the application of  $\bar{\epsilon}$  to obtain  $\sigma_{ij}$  and  $\epsilon_{ij}$
- Nonlinear **constitutive laws** can be applied. For instance, **plasticity of the matrix** for progressive failure analyses
- Defects, e.g., **voids**, can be included
- The **computational size of the RVE** determines the cost of the analysis
- In a multiscale scenario, **each Gauss point** of the macroscale model **has its own RVE**

Sun, C. T. and Vaidya, R. S. (1996), Composites Science and Technology, 56(2):171–179

Xia, Z. et al. (2003), International Journal of Solids and Structures, 40(8):1907–1921

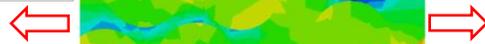
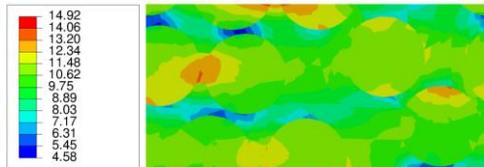
# Examples: Linear Elastic Homogenization

**Aim:** avoid 3D elements and favor multiscale analyses

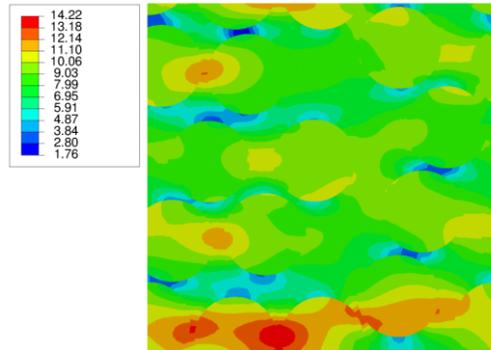


Normal transverse stress

**CUF-CW**  
DOF: 19,080  
Runtime: 18s

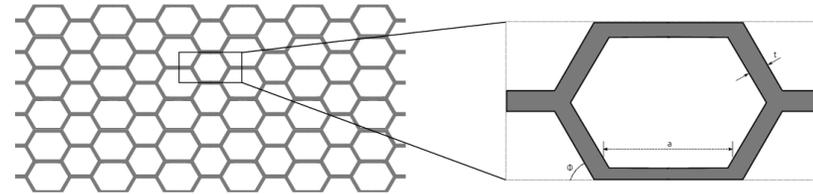


**3D FEM**  
DOF: 91,305  
Runtime: 324s

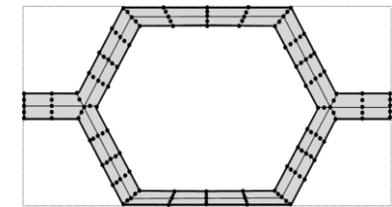


Effective moduli of periodical cellular structure

Architecture of hexagonal honeycomb



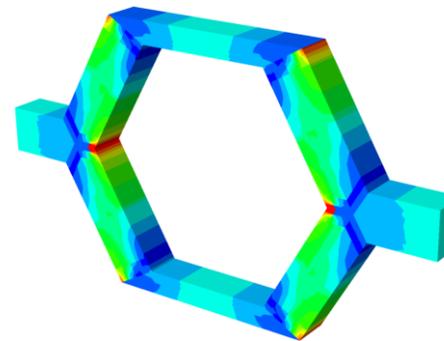
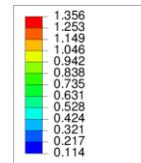
CW discretization of RVE – 18L9



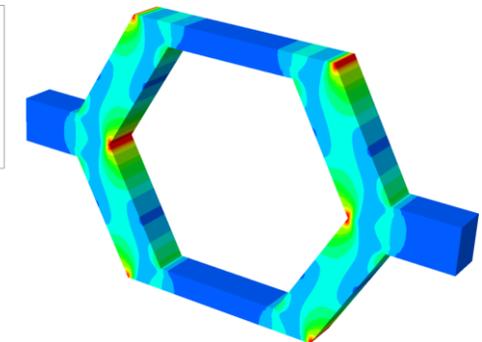
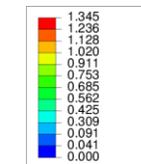
Predicted transverse Young's modulus

CUF-CW	FEM 3D	G-A MMM (Gibson et al.)
0.0504	0.0498	0.0485

3D von-Mises stress contour  $\sigma_{vm}$

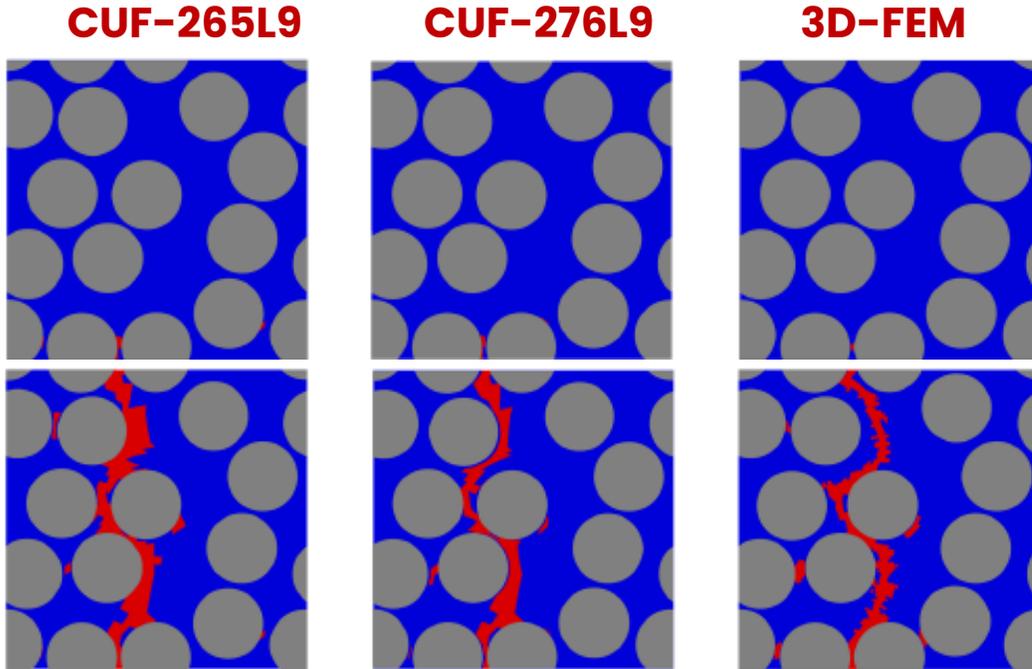


**CUF-CW | DOF: 6,993**

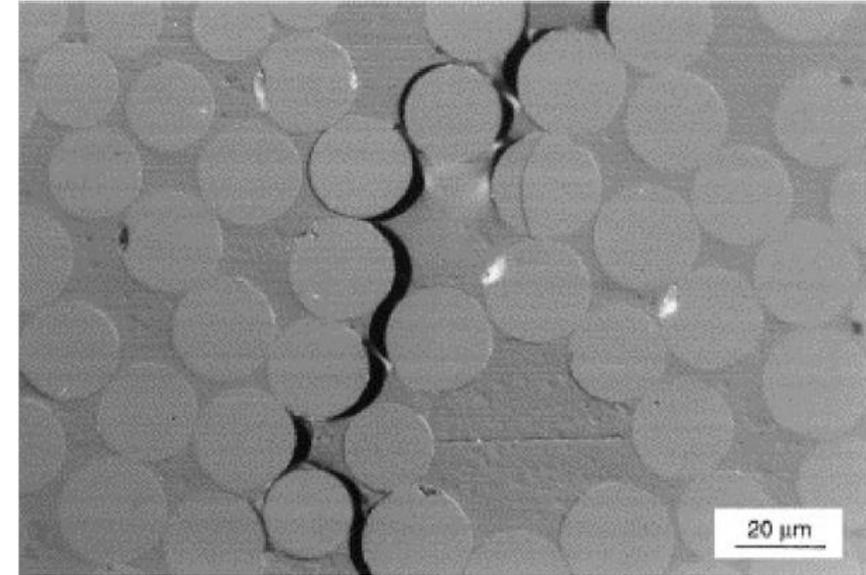


**3D FEM | DOF: 79,104**

# Examples: Progressive Damage



Matrix damage progression under applied transverse strain [1]



Formation of micro-cracks in static tension tests of cross-ply laminates [2]

Model	DOF	Time (min)
1D	23711	48
3D	91305	108

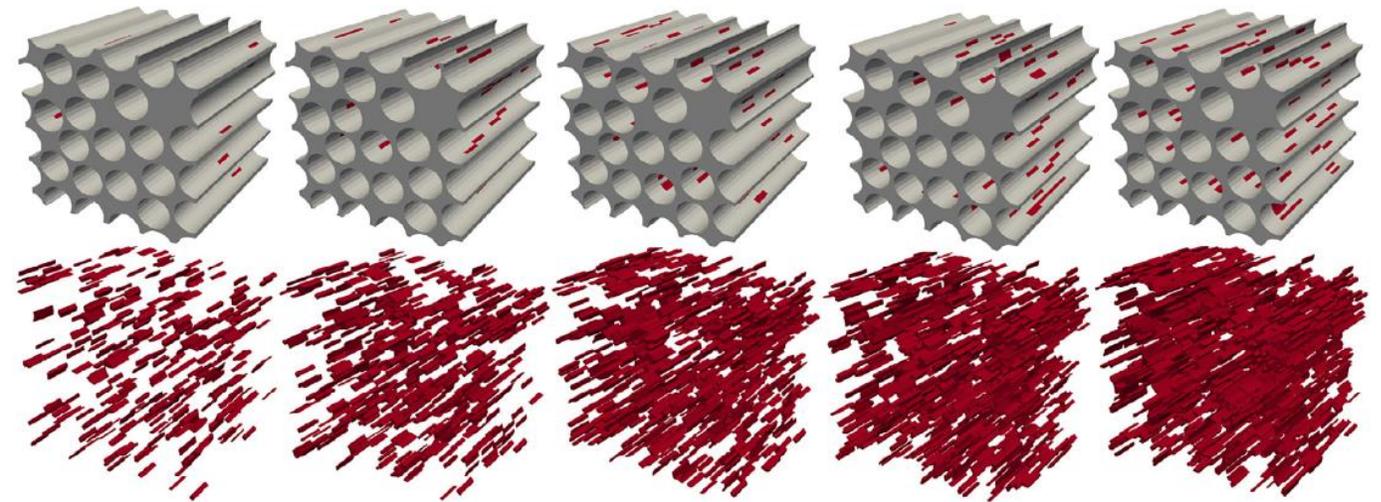
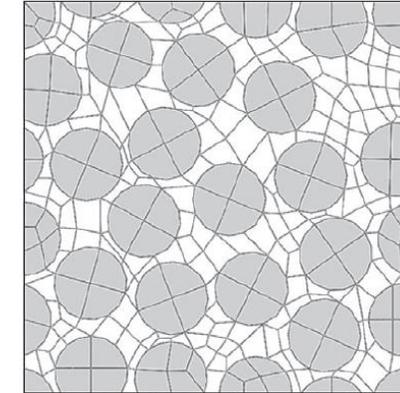
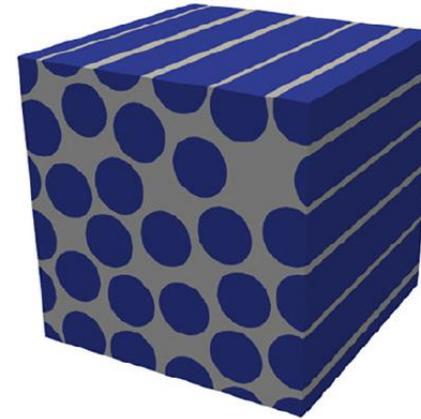
**~4x DOF**  
**~2.2x**  
**Time**

[1] Kaïeel I. et al. Journal of Applied Mechanics (2018), 85

[2] Gamstedt, E. K., & Sjögren, B. A. (1999). Composites Science and Technology, 59(2), 167-178

# Examples: Voids 1/3

- Randomly distributed voids in matrix
- Void volume fraction given as an input
- Use of distribution functions to randomly select matrix GP
- Material properties of selected GPs modified to exhibit void-like behaviour



1%

2%

3%

4%

5%

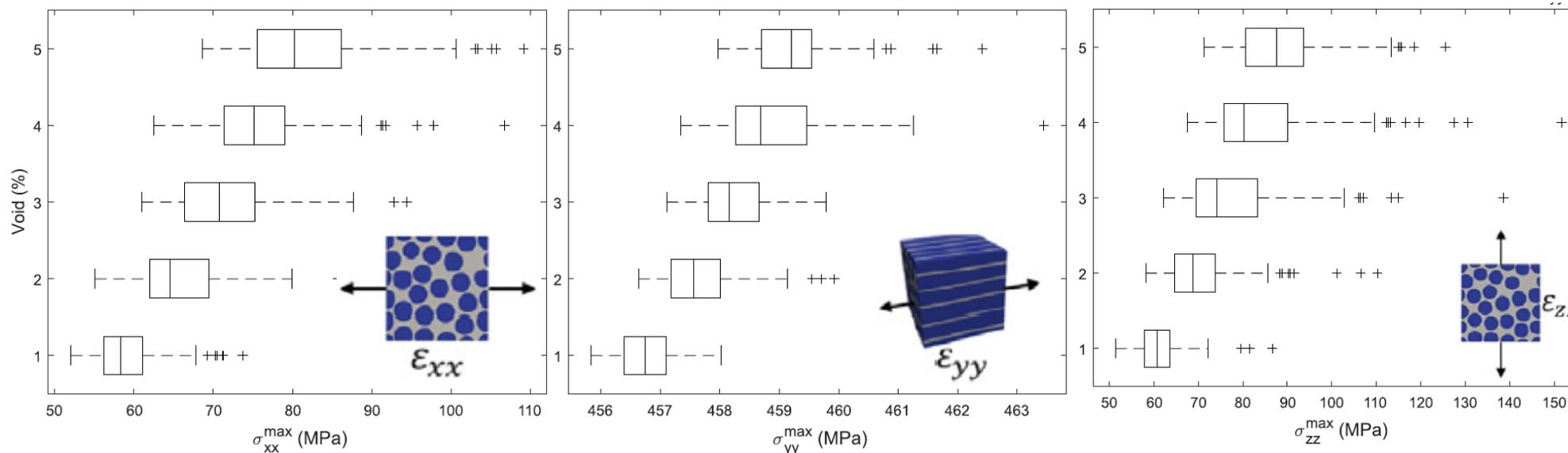
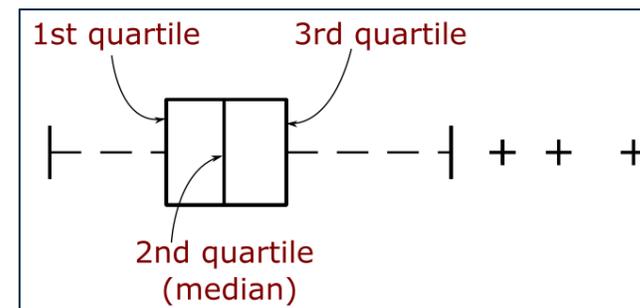
*RVE models with increasing void content [1]*

Carrera, E. et al. (2020). Composite Structures, 254, 112833

# Examples: Voids 2/3

## Homogenised Properties

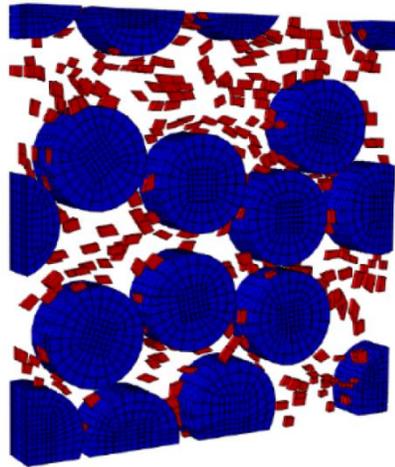
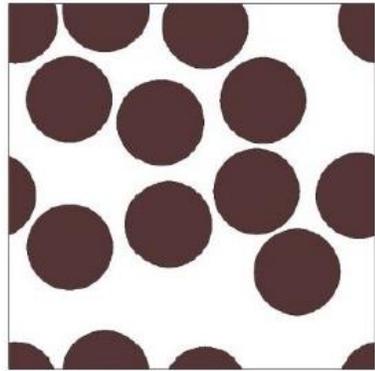
		Void content (%)				
		1	2	3	4	5
$\bar{x}$ (GPa)	$E_{11}$	135.846	135.809	135.772	135.734	135.697
	$E_{22}$	9.887	9.818	9.749	9.679	9.609
	$E_{33}$	9.836	9.766	9.696	9.625	9.555
	$G_{12}$	5.156	5.127	5.098	5.069	5.040
	$G_{13}$	4.993	4.964	4.935	4.906	4.878
	$G_{23}$	3.125	3.103	3.080	3.057	3.034
$s$ (MPa)	$E_{11}$	0.508	0.680	0.784	0.850	0.963
	$E_{22}$	1.470	2.174	2.231	2.910	3.108
	$E_{33}$	1.307	2.036	2.761	2.405	3.127
	$G_{12}$	1.064	1.673	1.835	1.979	2.064
	$G_{13}$	0.969	1.304	1.894	1.790	2.165
	$G_{23}$	0.456	0.580	0.647	0.790	0.868



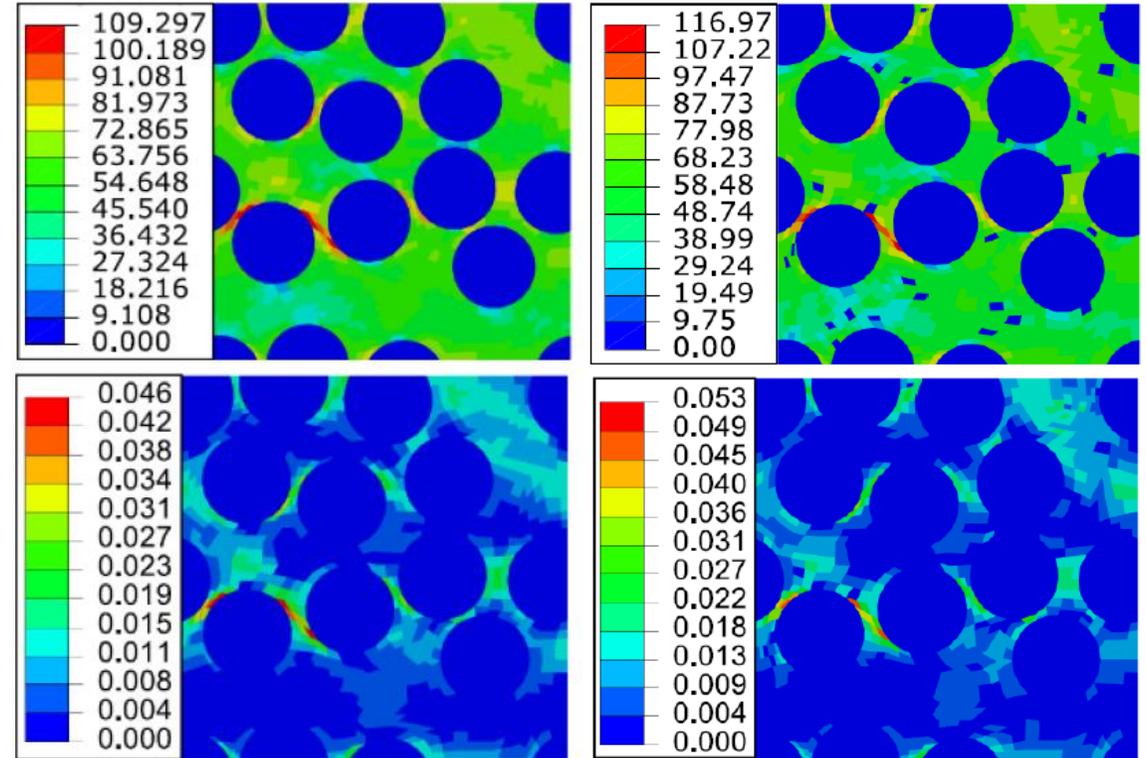
Carrera, E. et al. (2020). Composite Structures, 254, 112833

Maximum local stress components as a function of void content

# Examples: Voids 3/3



*1-D RVE model with 2% randomly distributed voids*



*Local von Mises stress (top) and eq. plastic strain (bottom, for applied -22 global strain)*

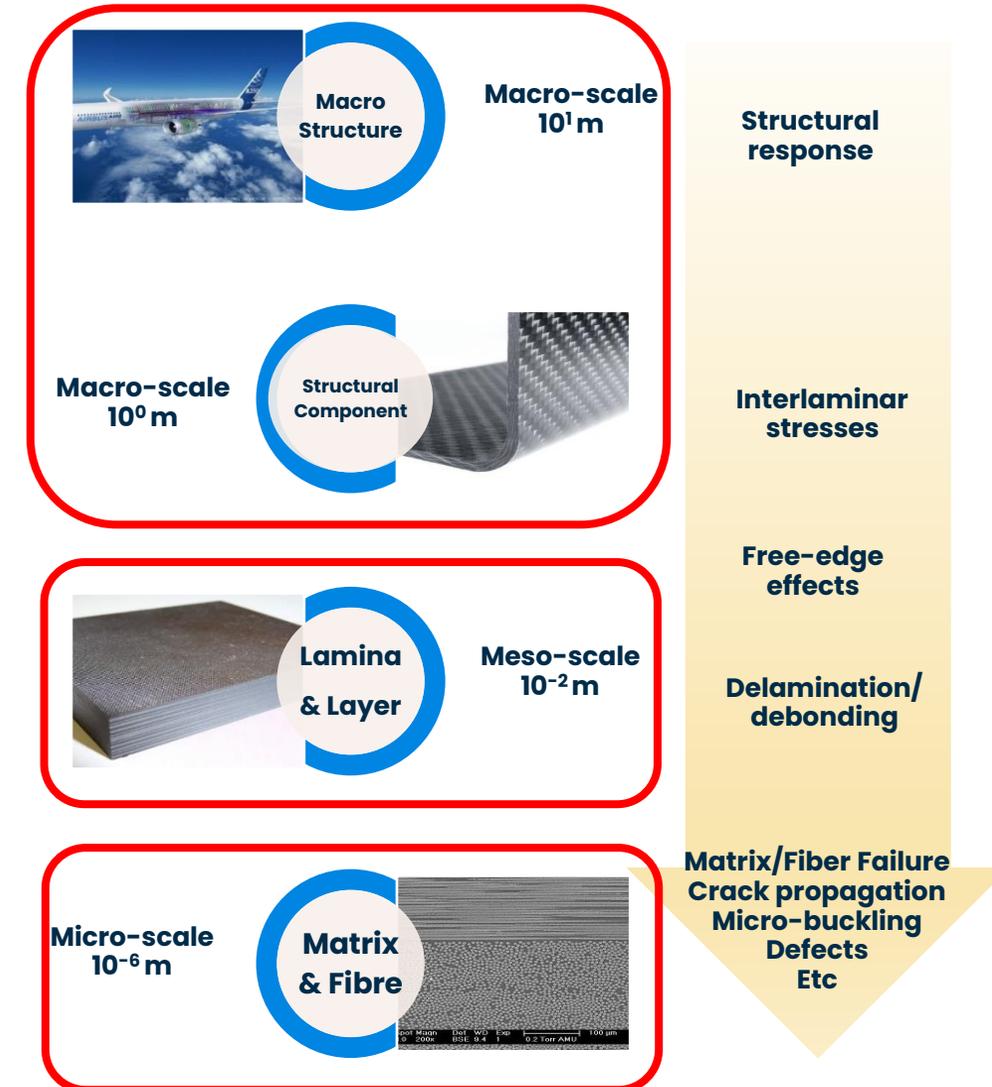
# PART II – Failure and Multiscale Analyses

## MULTISCALE

# Motivations

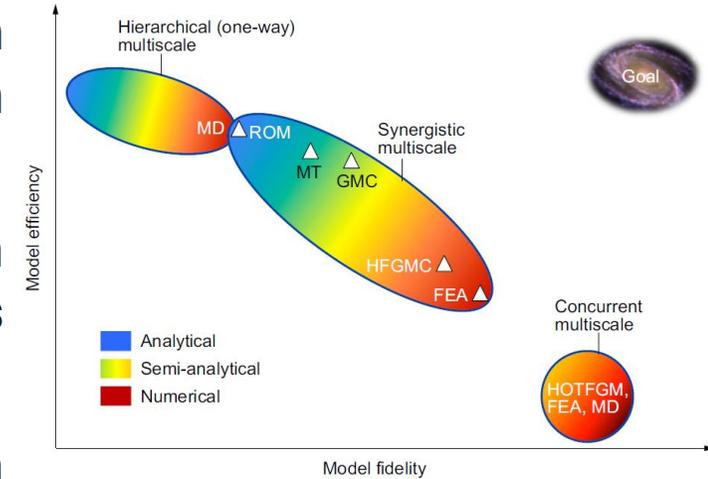
- Understanding the source of physical behavior at **lower scales** and **bridging** the effect accurately to **upper scales** can significantly **boost the fidelity** of such simulations
- Continuum-based constitutive modeling has proven to be well-suited for predicting the overall response of structures, modeling **localized phenomena** such as damage and failure propagation using such approaches remains questionable as such mechanisms are **heavily influenced by the underlying lower-scale features**
- Macroscale constitutive modeling assumes the **material point as a homogeneous** and **heterogeneity** such as inclusions, voids etc. are accounted through **implicit mathematical formulations**.
- Within a multiscale framework, the constitutive response at **a material point is interfaced** with a lower scale with **explicit heterogeneous definitions** through homogenization

Sullivan, R. W. and Arnold, S. M. (2010). An annotative review of multiscale modeling and its application to scales inherent in the field of ICME. In Models, Databases, and Simulation Tools Needed for the Realization of Integrated Computational Materials Engineering. ASM International.



# Approaches

- Hierarchical: based on **one-way coupling** with information passed **either bottom-up** (homogenization) **or top-down** (localization)
- Concurrent: **fully-coupled information interfacing** with bottom-up and top-down occur concurrently with **all scales handled simultaneously** in space and time
- Synergistic: fully coupled, **hierarchical in space, concurrent in time**
- As in micromechanics, there are **analytical, semi-analytical,** and **fully numerical** approaches
- **Most common fully-numerical** approaches are based on finite element models applied at both scales, often referred to as **FE<sup>2</sup> method**

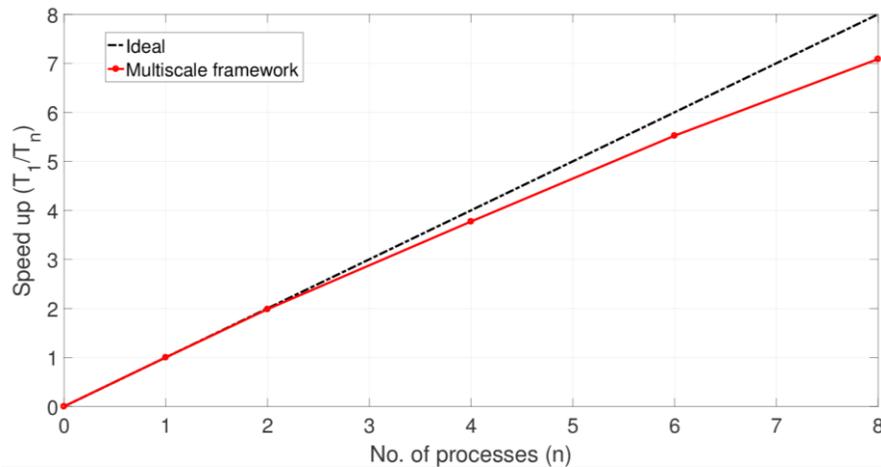


Aboudi, J. et al. (2013),  
Micromechanics of Composite  
Materials: A Generalized Multiscale  
Analysis Approach. Elsevier

Ladevèze, P., et al. (2001),  
International Journal for Numerical  
Methods in Engineering, 52(1-  
2):121-138

# Multiscale in MUL2

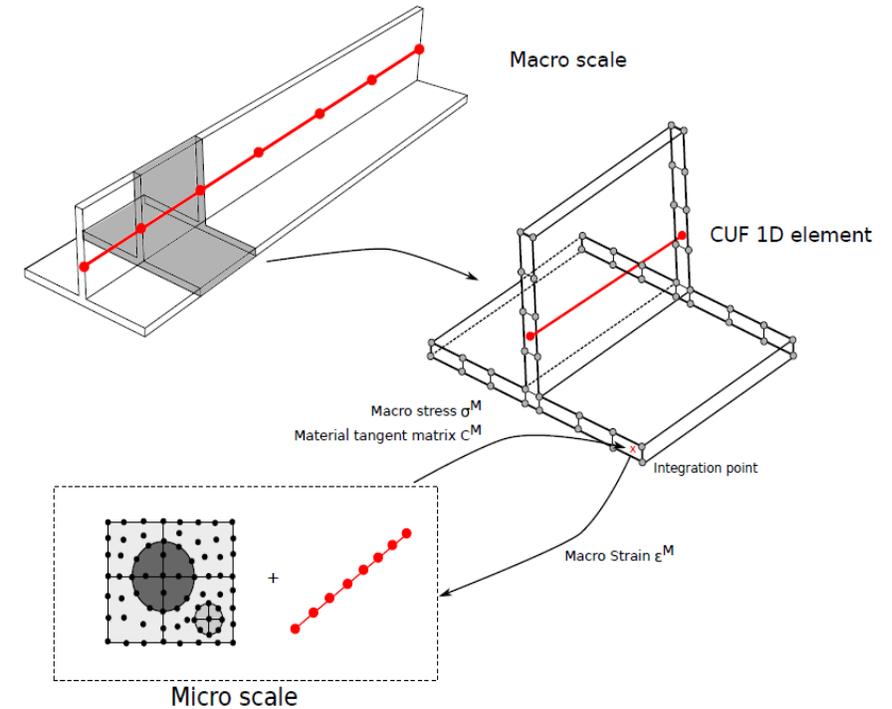
- **Synergistic** multiscale framework, potentially concurrent
- RVE at **microscale via 1D CUF**
- **1D or 2D CUF** models for **macroscale**
- Highly scalable due to **parallel implementation** (Hybrid OpenMP + MPI)



*Speed-up Vs. No. of processors*

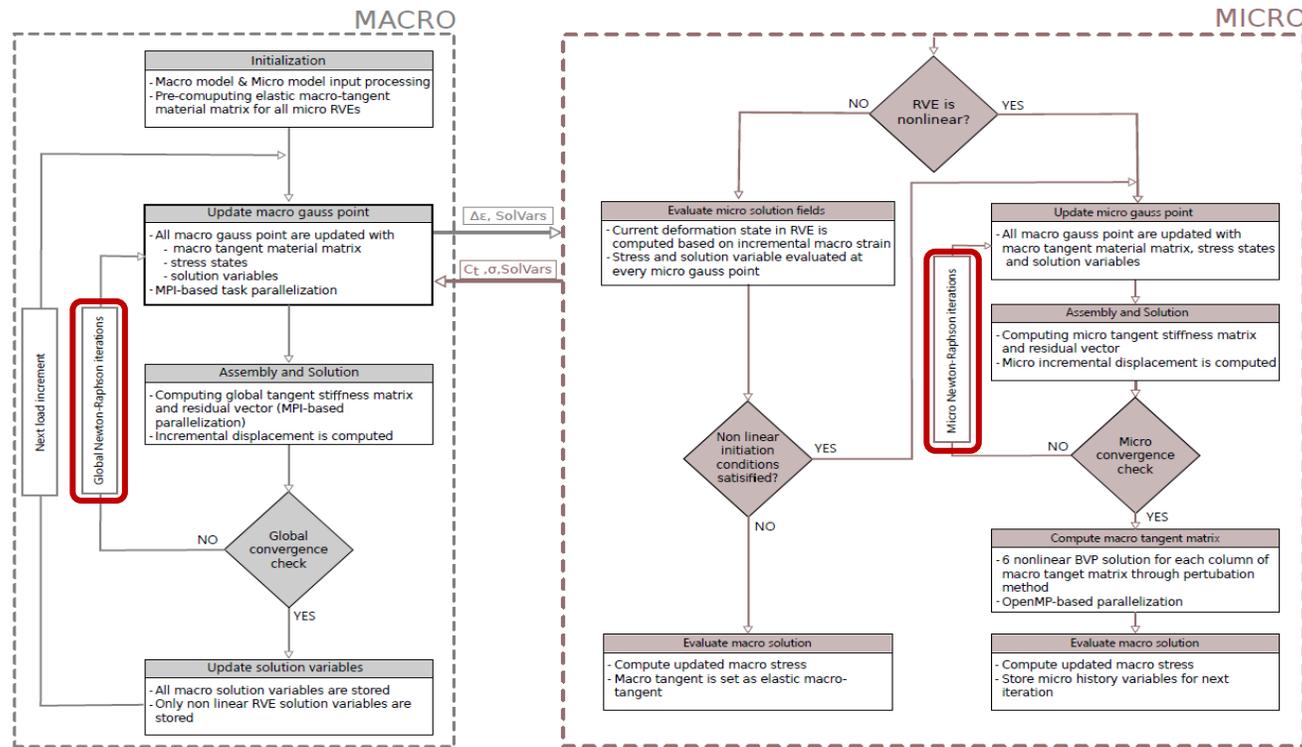
Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028



*Schematic representation of the multiscale analysis procedure*

# Multiscale in MUL2/Implicit



- **Fully implicit** multiscale framework
- Nonlinear solver based on Newton-Raphson at **both scales**
- Typical performances shown in the Table below
- Composite **open-hole specimen**, material nonlinearity

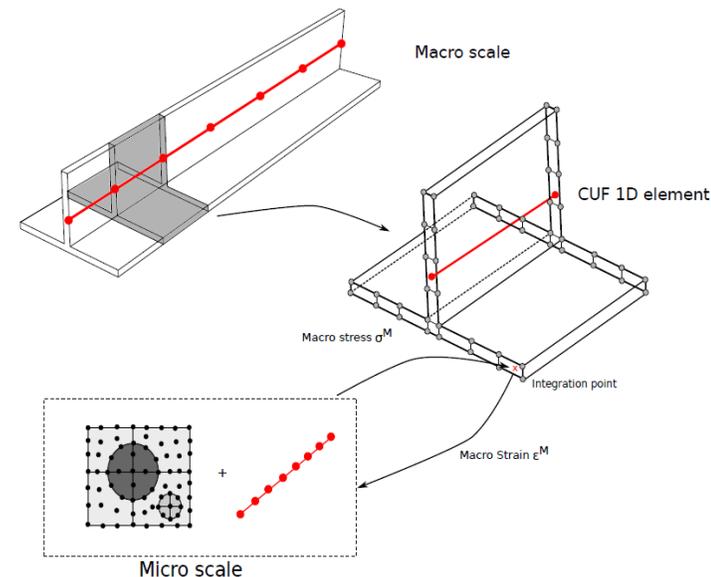
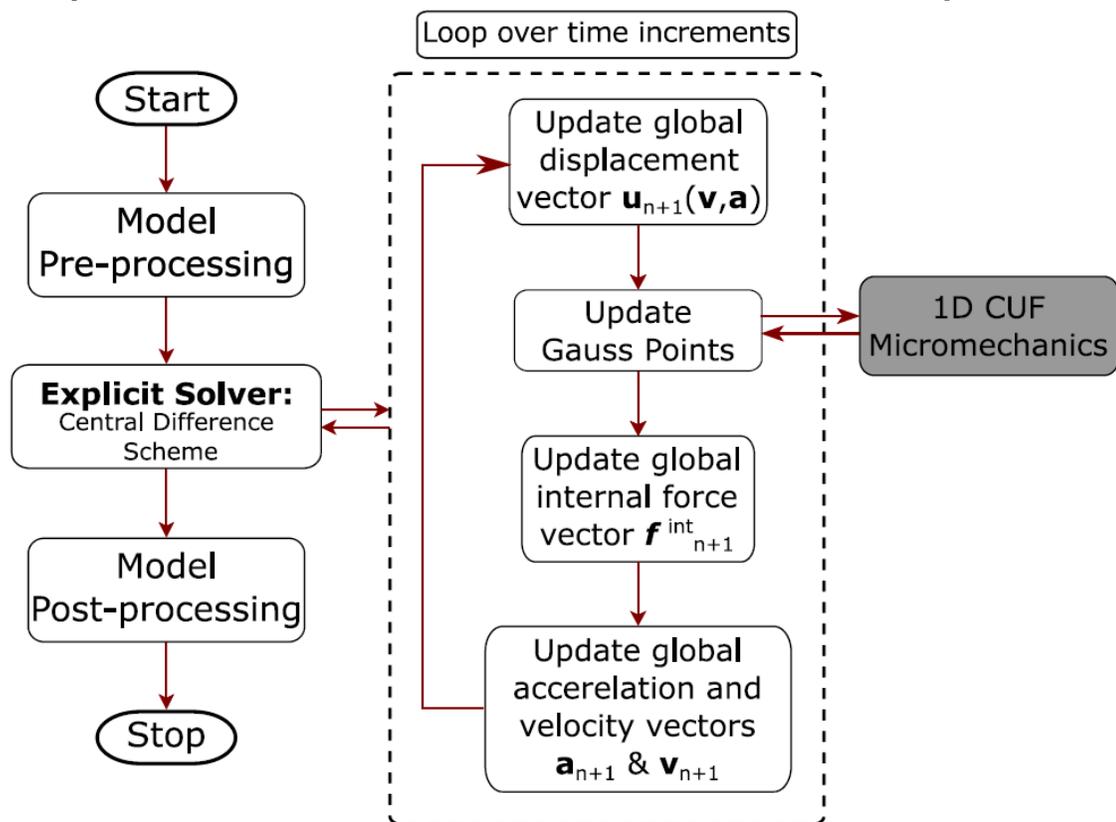
Model	Memory Requirement (Gb)	Time
1D /1D	2.02	2 hrs
3D coarse/1D	7.28	5.5 hrs
3D refined/1D	8.75	8.5 hrs

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

# Multiscale in MUL2/Explicit

1D CUF-Micromechanics toolkit based on Newton-Raphson for micromechanical analysis



*Use of explicit solver improves numerical convergence – feasibility of damage analysis*

Model	Time
2D /1D	3.5 hrs
3D coarse/1D	6.0 hrs
3D refined/1D	8.5 hrs

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

# Example: stiffness prediction of laminates

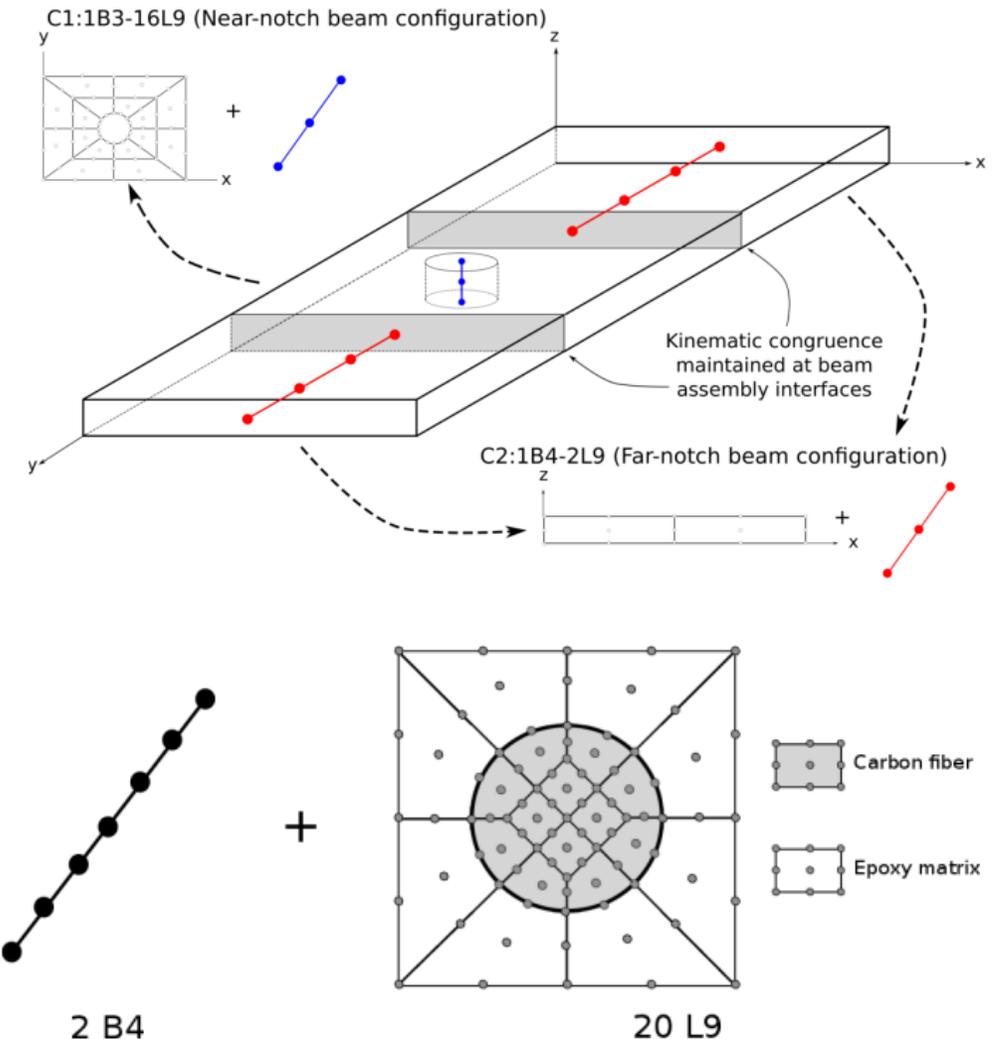
- Fiber volume fraction: 65%
- **Open-hole** specimen
- Macro specimen interfaced with a **square packed RVE**
- **Tensile stiffness prediction** for three layups

	Experiment al	MAC/GMC	CUF 1D/1D
$[0\backslash 45\backslash 90\backslash -45]_{2s}$	48.3	49.1 (1.66%)	49.4 (2.28%)
$[60\backslash 0\backslash 60]_{3s}$	48.8	48.9 (0.20%)	50.44 (3.36%)
$[30\backslash 60\backslash 90\backslash 30\backslash -60]_{3s}$	32.4	33.7 (4.01%)	33.25 (2.62%)

All units in GPa. Quantities in parenthesis represent error with respect to experimental result

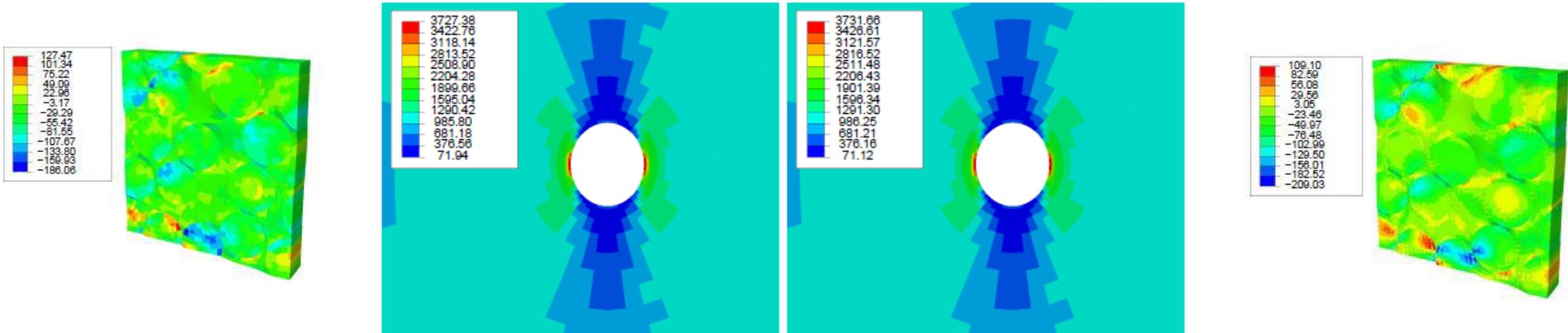
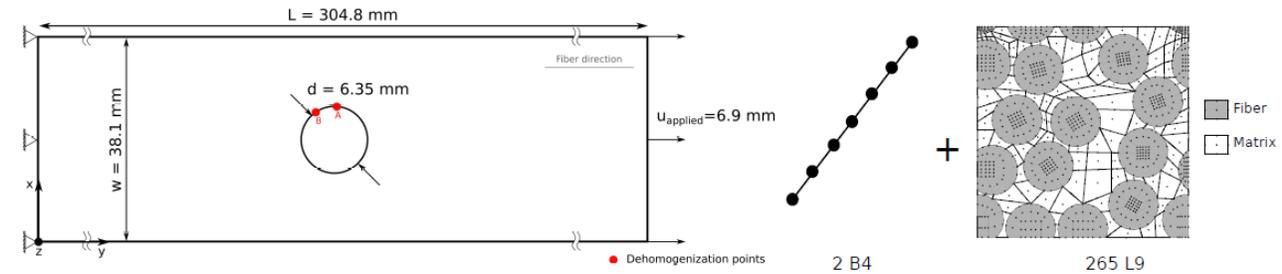
Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028



# Example: linear multiscale analysis with randomly distributed RVE

- Two multiscale models
  - 1. **1D/1D** (Macro and micro model: CUF);
  - 2. **1D/3D** (Macro: CUF beam and micro: 3D)
- 1D (**left**) and 3D (**right**)

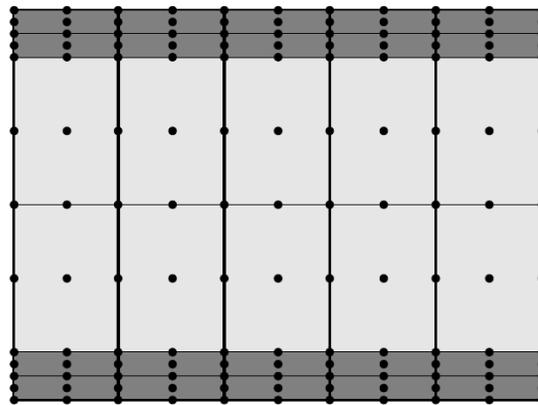
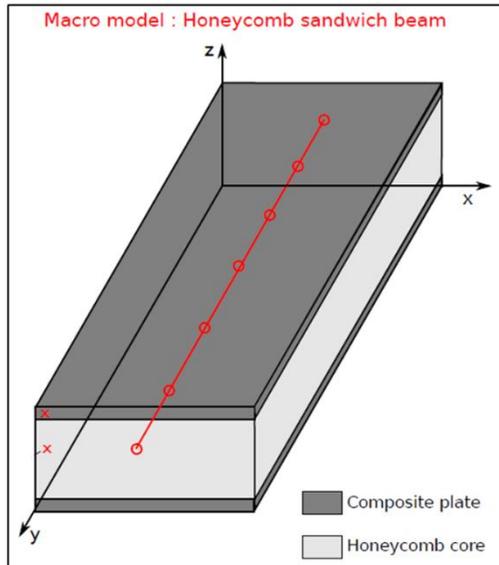
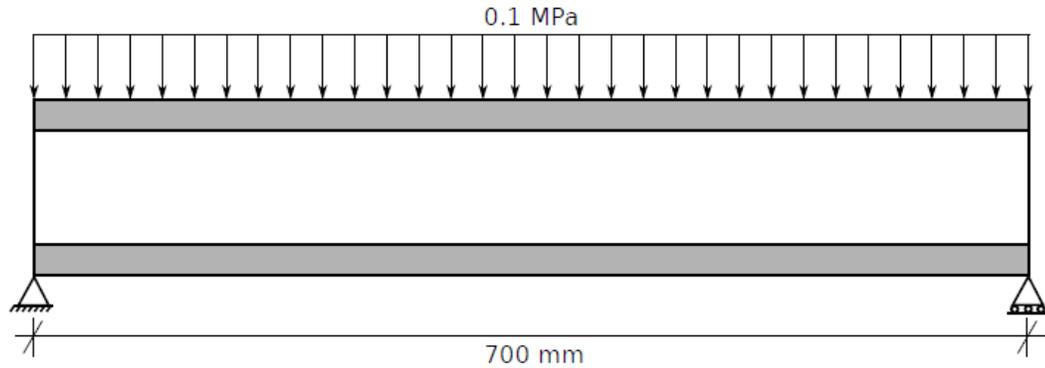


Model	Time (s)
1D /1D	10
1D/3D	42

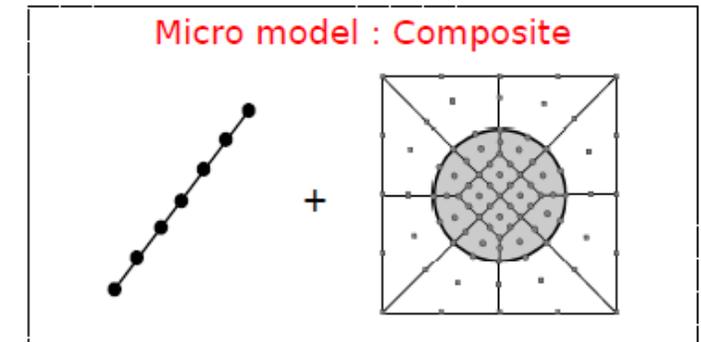
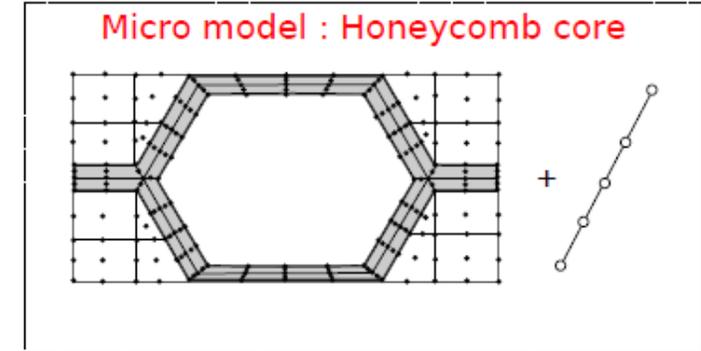
Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

# Example: linear multiscale analysis of a honeycomb sandwich plate 1/2



Macroscale cross-section mesh

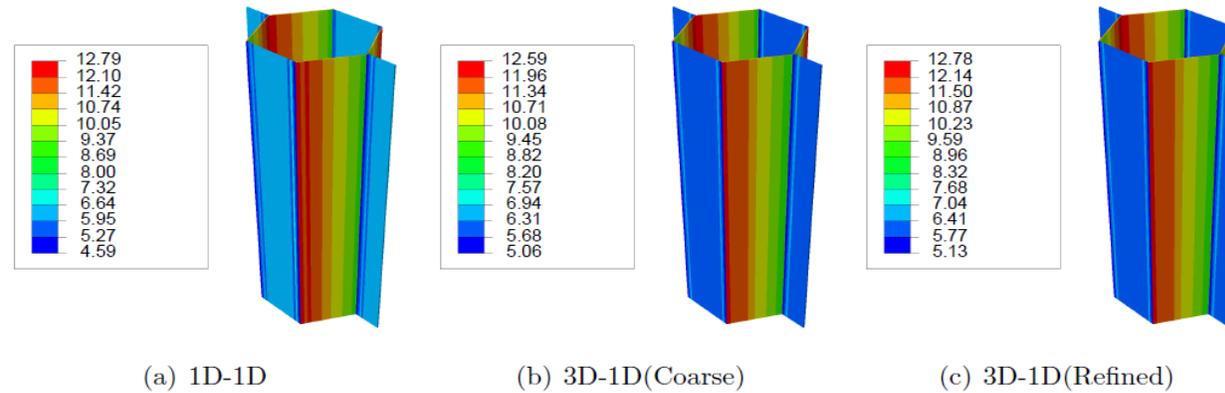


RVE discretization at the microscale

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

# Example: linear multiscale analysis of a honeycomb sandwich plate 2/2



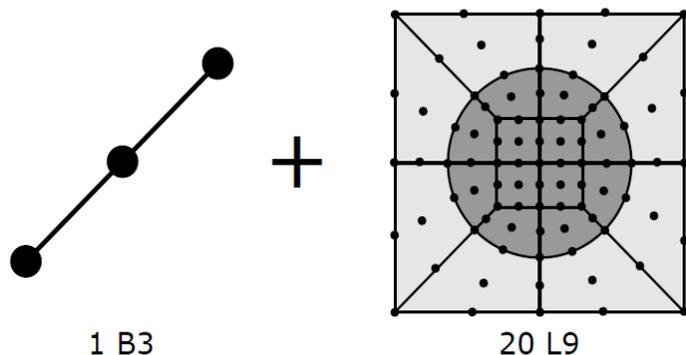
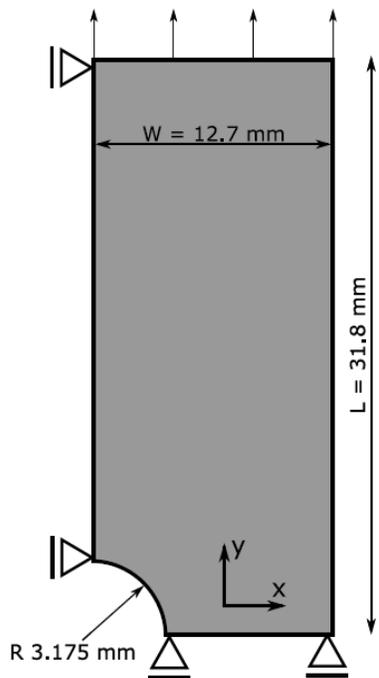
*Local von Mises stress field*

Analysis	Mesh	DOF	Gauss Pts.	No. of CPUs	Analysis Time (s)		Memory required for macro – GP (GB)
					Without Local Fields	With Local Fields	
1D – 1D	36 L9 – 15 B4	19,734	16,200	1	14.9	64.0	2.95
3D – 1D (coarse)	12,500 linear brick	43,758 <b>(~2.2x)</b>	100,000	4	27.9 <b>(~1.9x)</b>	181.2 <b>(~2.8x)</b>	19.8 <b>(~6.7x)</b>
3D – 1D (refined)	20,000 linear brick	69,498 <b>(~3.5x)</b>	160,000	4	44.0 <b>(~2.9x)</b>	287.0 <b>(~4.5x)</b>	31.6 <b>(~10.7x)</b>

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

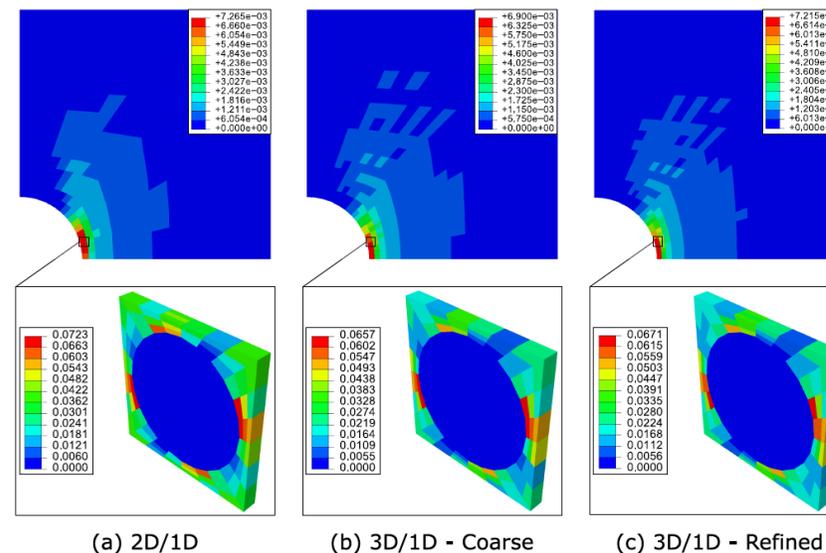
# Example: elastoplastic response of open-hole tensile coupon



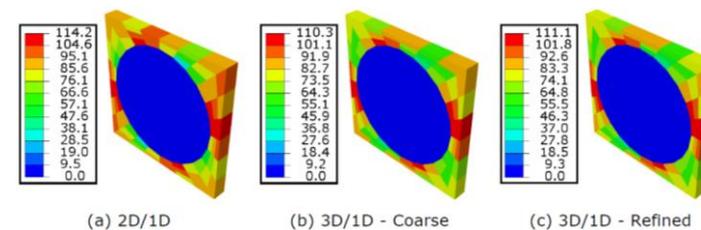
*MICROSCALE: Square-packed RVE with 1B3/20L9 discretisation*

*MACROSCALE: Open-hole [0/90/0] tension coupon (1/4 model)*

Model	Time
2D /1D	<b>3.5 hrs</b>
3D coarse/1D	<b>6.0 hrs</b>
3D refined/1D	<b>8.5 hrs</b>



*Inelastic strain fields at macro- and micro-scales*



*Von Mises stresses at the microscale*

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4029-4041

Kaleel I. et al. (2019), AIAA Journal, 57(9), pp. 4019-4028

# NASA Multiscale Analysis Tool (NASMAT)

- **New** state-of-the-art **multiscale framework** developed at the **NASA Glenn Research Center** for use on **high performance computing systems**
- Already **includes several** semi-analytical micromechanics **theories**, constitutive and **damage models**
- Uses recursive data structures and subroutines to facilitate analyses utilizing an **arbitrary number of length scales**
- Several application program interfaces developed to support **interoperability with third party software**
- The **MUL2 micromechanics** is now one of the tools **available in NASMAT**

Main features:

- An **Abaqus** GUI generates CUF/**MUL2** meshes
- NASMAT can **combine CUF/MUL2 models** with **any material model** available in its libraries
- A library of **predefined CUF architectures** implemented in NASMAT (Square-packed, square-packed with coating, hex-packed, hex-packed with coating)

Aboudi J., Arnold S., Bednarczyk B. (2021), Practical Micromechanics of Composite Materials, Butterworth-Heinemann

# Further Topics and Reading

- Geometrical Nonlinearity  
Pagani, A. et al. (2019), AIAA JOURNAL, Vol. 57, pp. 3524-3533
- Multifield problems  
Nali P. et al., International Journal for Numerical Methods in Engineering (2011), 85, pp. 869-919;  
Carrera E., AIAA Journal (2005), Vol. 43, N° 10, pp. 2232-2242;  
Carrera E. and Nali P., International Journal for Numerical Methods in Engineering (2009), 80, pp. 403-424
- Node-Dependent-Kinematics and Global/Local  
Zappino, E., Carrera, E., (2018) AIAA Journal, 56:4, pp 1647-1661
- Rotordynamics  
Filippi, M. et al. (2019), JOURNAL OF THE AMERICAN HELICOPTER SOCIETY, Vol. 64, pp. 1-10
- Virtual Manufacturing  
Zappino, E. et al. (2020), COMPOSITE STRUCTURES, Vol. 241
- Axiomatic/Asymptotic Method, Best Theory Diagrams, Neural Networks  
Carrera E., Petrolo M., AIAA Journal (2010), Vol. 48, N° 12, pp.2852-2866  
Petrolo M., Carrera E., (2019), Advanced Modeling and Simulation in Engineering Science, 6(4), pp. 1-23  
Petrolo, M., Carrera, E., (2021), COMPUTERS & STRUCTURES, Vol. 244

Full list: <http://www.mul2.polito.it/index.php/publications/journal-articles>

# Composite materials: Modeling, Processing, and Characterization

Structural Modeling and Analysis

8-9 June 2021, 14:00-16:00

Speaker: Marco Petrolo, [www.mul2.com](http://www.mul2.com)

Contributors:

Prof. Erasmo Carrera, Dr. Ibrahim Kaleel, Dr. Manish H. Nagaraj

# Processing methods and simulation of composites

2021-06-07

Navid Zobeiry, PhD (navidz@uw.edu)  
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Materials Science & Engineering  
University of Washington



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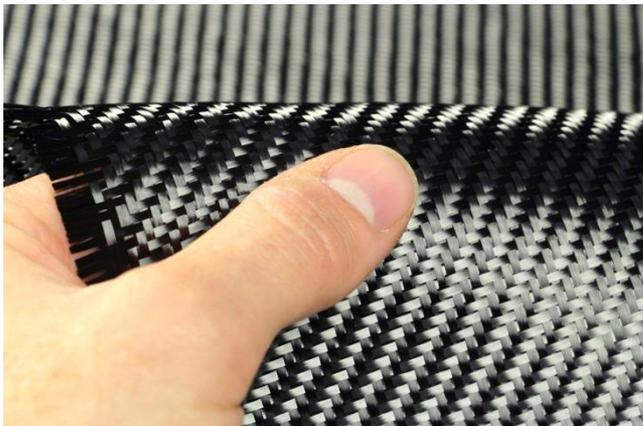
**W**

# What are Advanced Composites?

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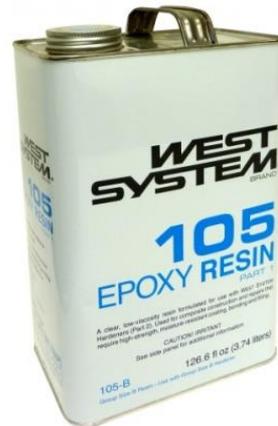
- Composite Materials or 'Composites' are:
  - *Made from two or more constituent materials with different properties (mechanical, chemical ...)*
  - *Remain separate and distinct within the finished structure*
- In typical industrial applications, one material is much stronger than the other one. The weaker material acts as a binding agent.

## Example of Advanced Composites (or composites)

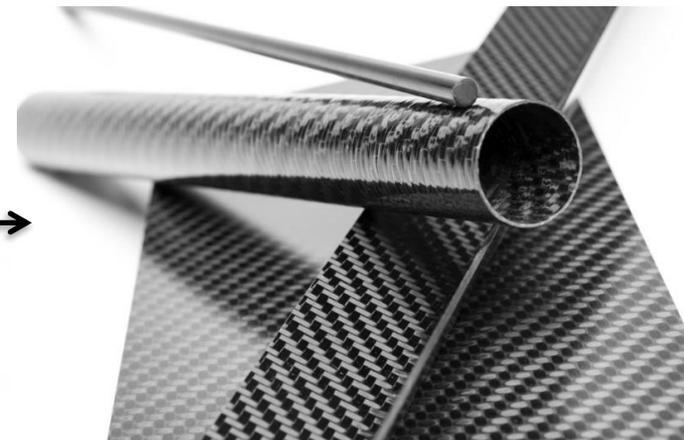


Advanced Fibers  
(example Carbon fiber)

+



Matrix  
(example Epoxy)



Advanced Composite parts

# Introduction: A Cooking Analogy

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- Composites processing and cooking have a lot in common
- We'll use a cooking analogy to go through a typical manufacturing process



Cooking food



Processing composites

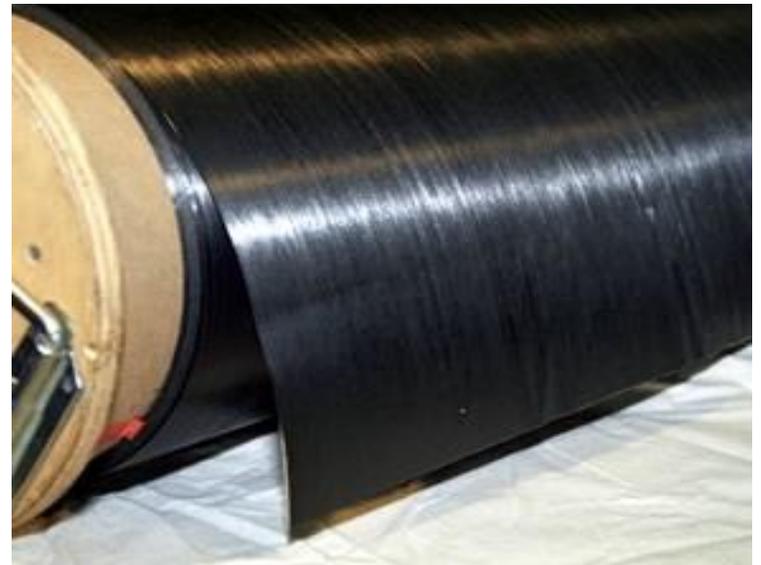
# Introduction: A Cooking Analogy

---

Choose the ingredients



Choose the constituent materials



# Introduction: A Cooking Analogy

---

Choose the cooking method



Choose the process



# Introduction: A Cooking Analogy

---

Choose the cookware



Choose the tool



# Introduction: A Cooking Analogy

---

Prepare the ingredients



Prepare the materials



# Introduction: A Cooking Analogy

---

Grease the pan



Apply release agent



# Introduction: A Cooking Analogy

---

Mix ingredients and place in pan



Deposit material on tool



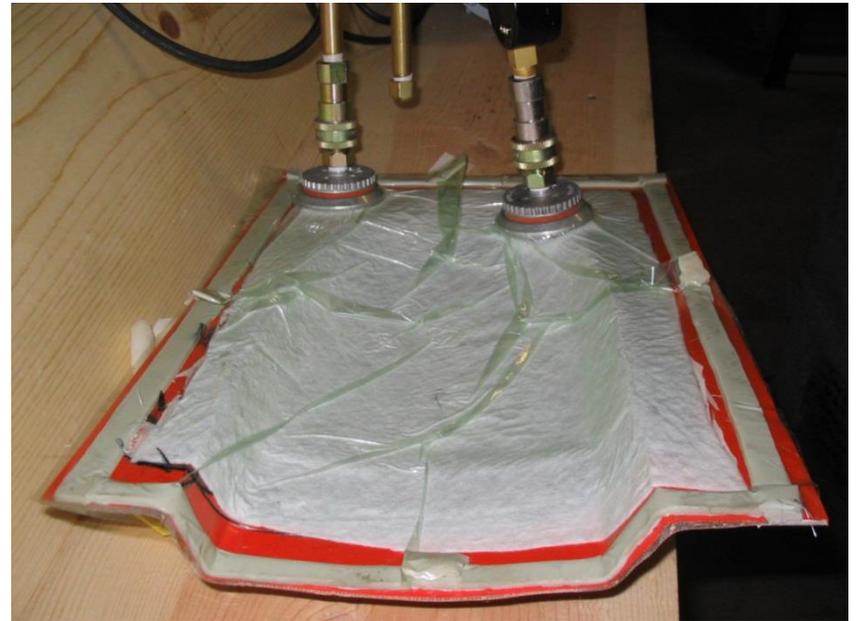
# Introduction: A Cooking Analogy

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Cover with foil



Apply vacuum bag



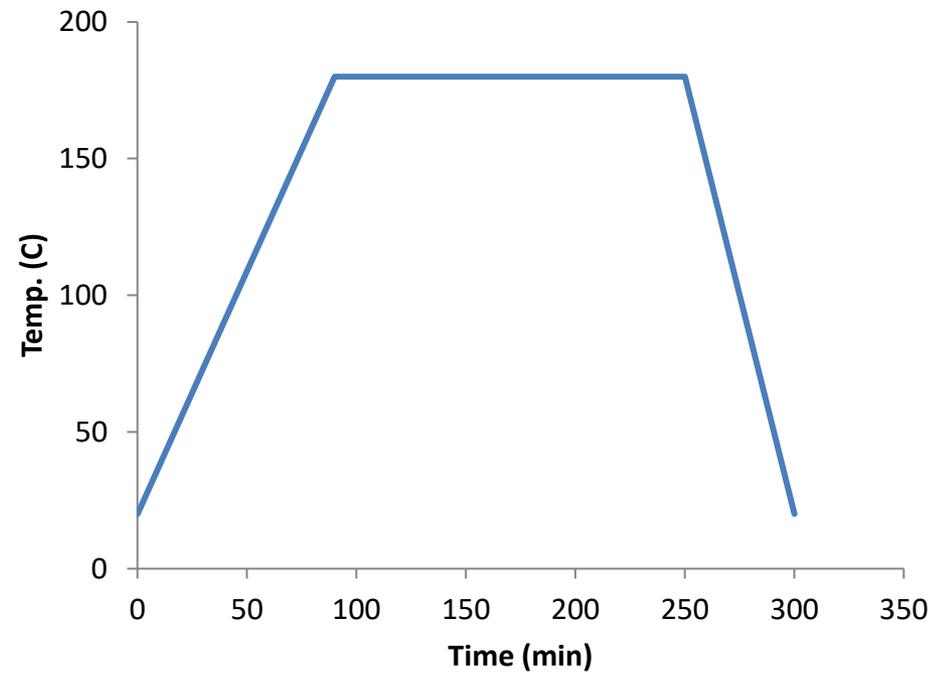
# Introduction: A Cooking Analogy

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Bake in oven



Apply cure cycle



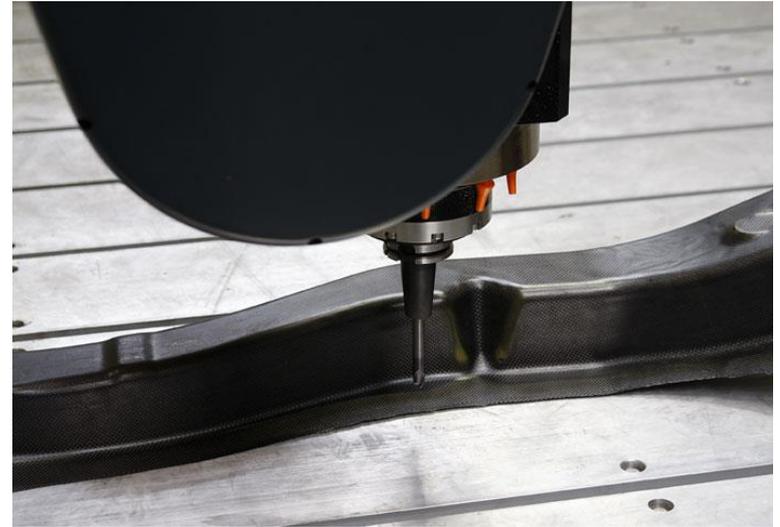
# Introduction: A Cooking Analogy

---

Cut and serve



Demould and trim  
(post processing)



# Composites Manufacturing

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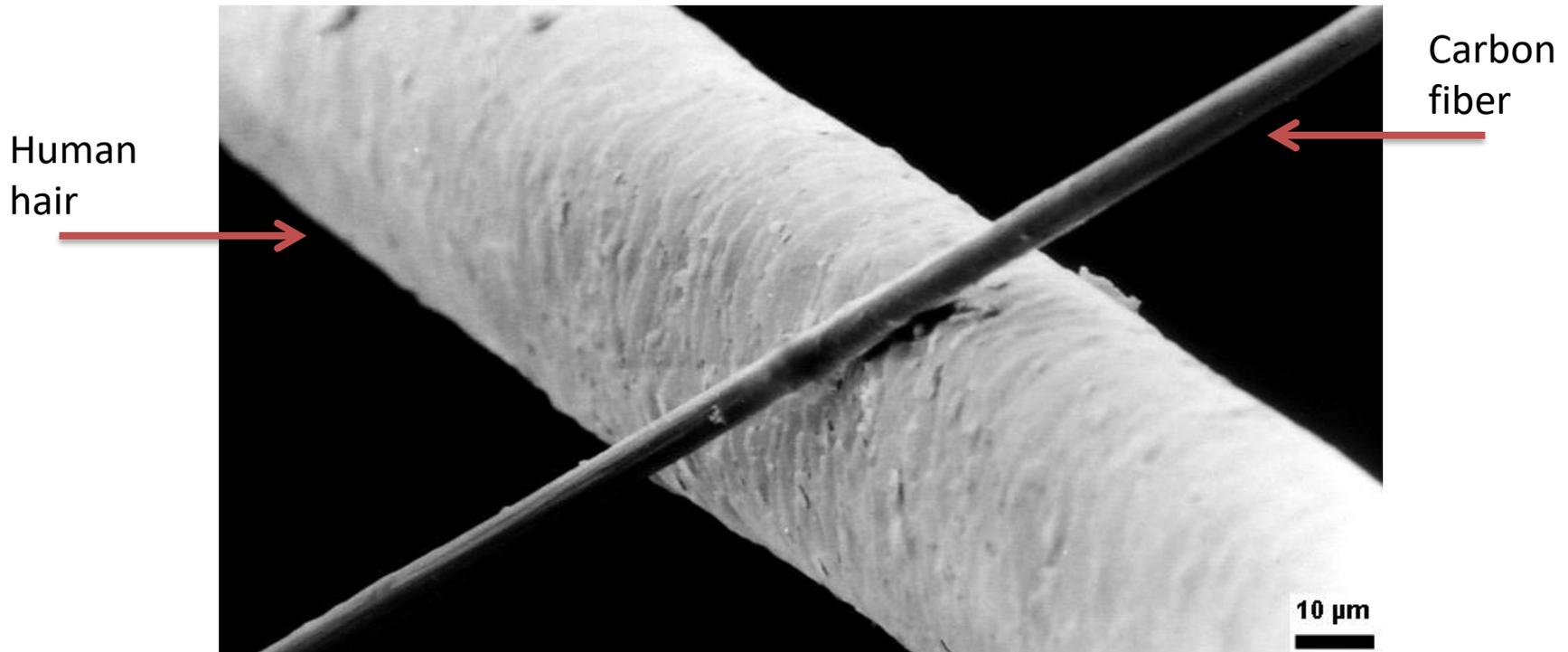
- Raw material
  - Fiber
  - Resin
  - Prepreg
  - ...
- Material deposition
  - Hand lay-up
  - Automated fiber placement (AFP)
  - Liquid composite molding (LCM)
  - ...
- Curing process
  - Autoclave
  - Out-of-autoclave
  - ...



# Advanced Fibers

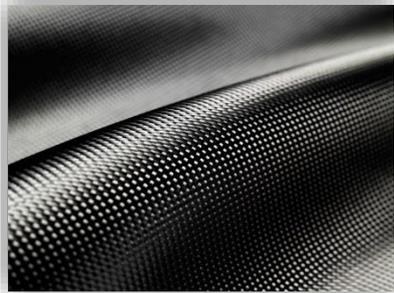
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- Advanced composite materials are characterized by high performance fibres:
  - Small diameter to reduce the probability of imperfections
  - High aspect ratio to effectively transfer the load
  - High flexibility for ease of processing



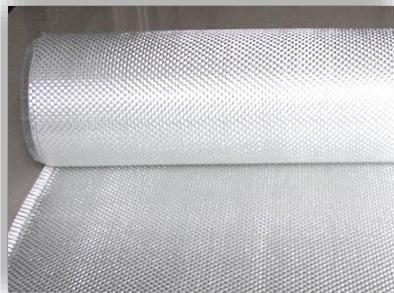
# Commonly used Advanced Composites

Carbon Fiber



CFRP

Glass Fiber



GFRP

## Thermoset Resins

- Epoxy
- Vinyl ester
- Polyester
- ...



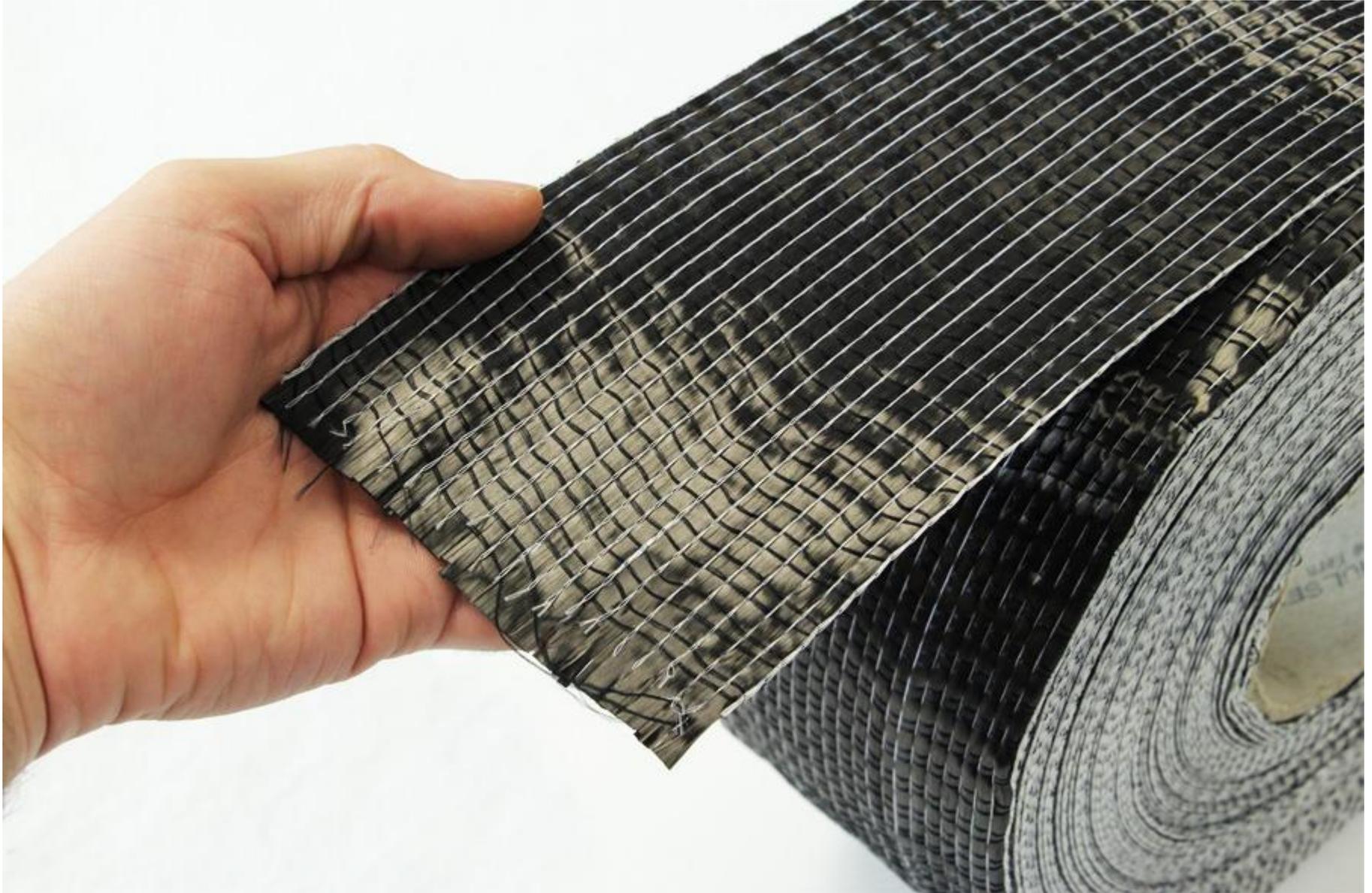
+

## Thermoplastic Resins

- Polypropylene
- PEI
- PEEK
- ...

# Fiber Architecture: Unidirectional

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# Fiber Architecture: Plain Weave Fabric

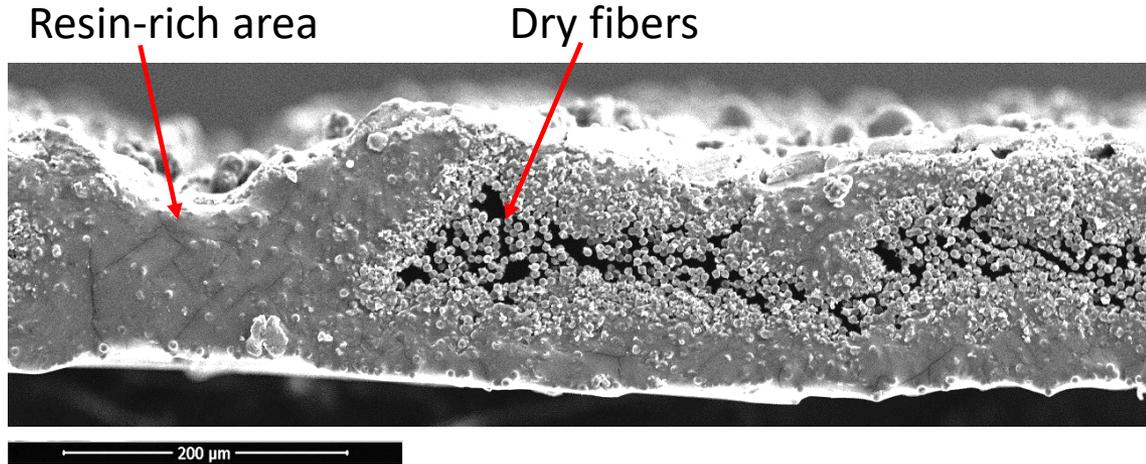
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- Consists of tows that are interlaced in an alternating fashion (over one, under one, over one, under one...)

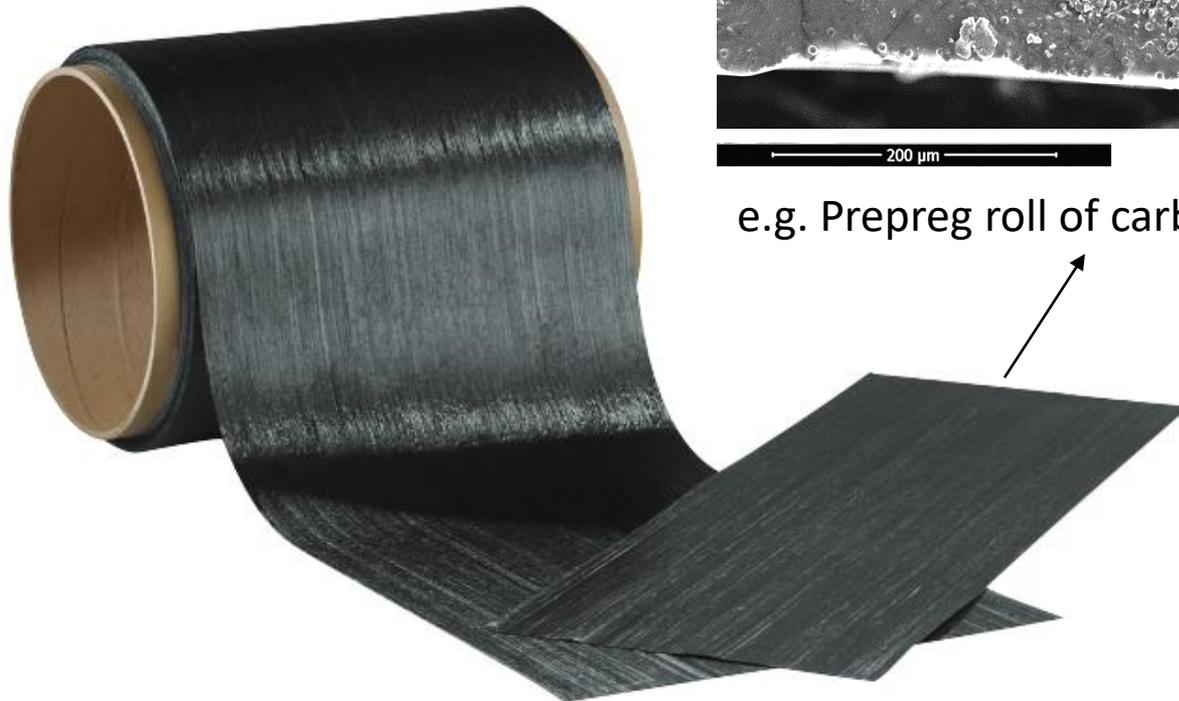


# Raw Material: Prepreg

- Prepreg material refers to fiber that is already combined with resin



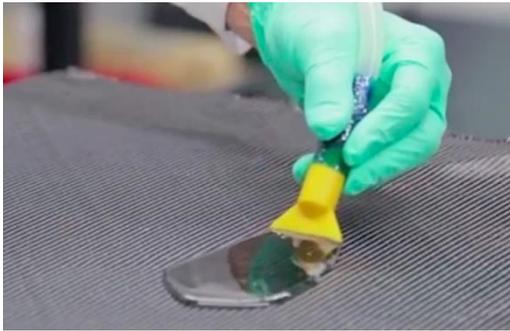
e.g. Prepreg roll of carbon fiber (60%) and epoxy (40%)



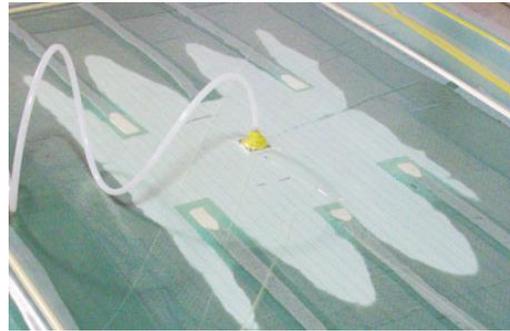
# Material Deposition Methods

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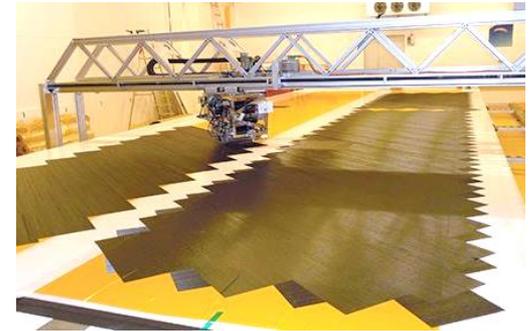
- There are three main methods to combine resin and fiber to deposit onto a tool:
  - **Wet Processes:** Mixing liquid resin and dry fiber directly in the mold
  - **Liquid Composite Molding (LCM) processes:** Infusing dry fabric with resin using a pressure gradient
  - **Prepreg Processes:** Laying-up using prepreg where resin and fiber are already combined



Wet Process



LCM



Prepreg Process

# Main Processing Methods

## Autoclave Processing



## Out-of-Autoclave (OOA) Processing

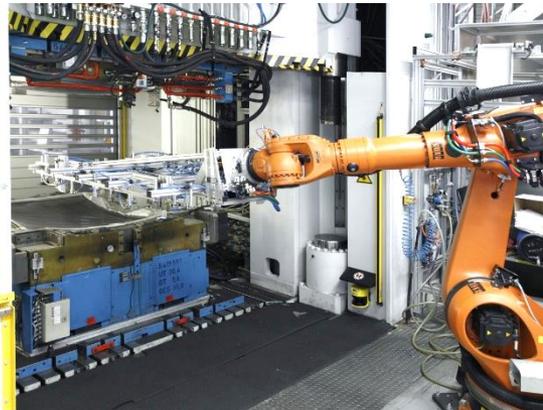
### Oven



### Microwave



### Hot press



### Heated tools



# Autoclave

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- Essentially, an autoclave is a heated pressure vessel
- Typical max pressures: ~7-10 bars
- Typical operating temperatures: ~200-400 °C
- Autoclaves are utilized where the **highest of material performance** standards are required such as void contents of less than 2%.
- It takes a lot of time and energy to produce parts using the Autoclave which make the end products **very expensive**.



# Autoclave: Major Cost in Aerospace



Boeing Autoclave for 787 fuselage



Boeing Autoclave for 777x Wing



Mitsubishi Autoclave for 787 wing

# OOA: Microwave Curing

- GKN Aerospace has developed microwave curing processing.
- An 80 percent **reduction in energy** use compared to autoclave cure, and a 40 percent **shorter cycle-time** was achieved.
- The **end-part quality is lower** than the autoclave part.



microwave oven-cured  
section of C-channel.



# OOA: Oven Curing

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- In oven curing, parts are vacuumed bagged and cured in an oven with **1 atmospheric pressure** compared to 7-10 bars in an autoclave. The process is extremely **sensitive to process parameters**.
- The Lockheed Martin X-55 Advanced Composite Cargo Aircraft (ACCA) is a transport aircraft which was built using a specific prepreg built for **oven curing**.



# OOA: Oven Curing



The Advanced Composite Cargo Aircraft's fuselage comprises eight pieces molded from OOA-cured MTM-45 prepreg

# OOA Processing Challenges

## Bombardier Learjet 85: Advanced Full Composite Structure



Full composite materials airframe

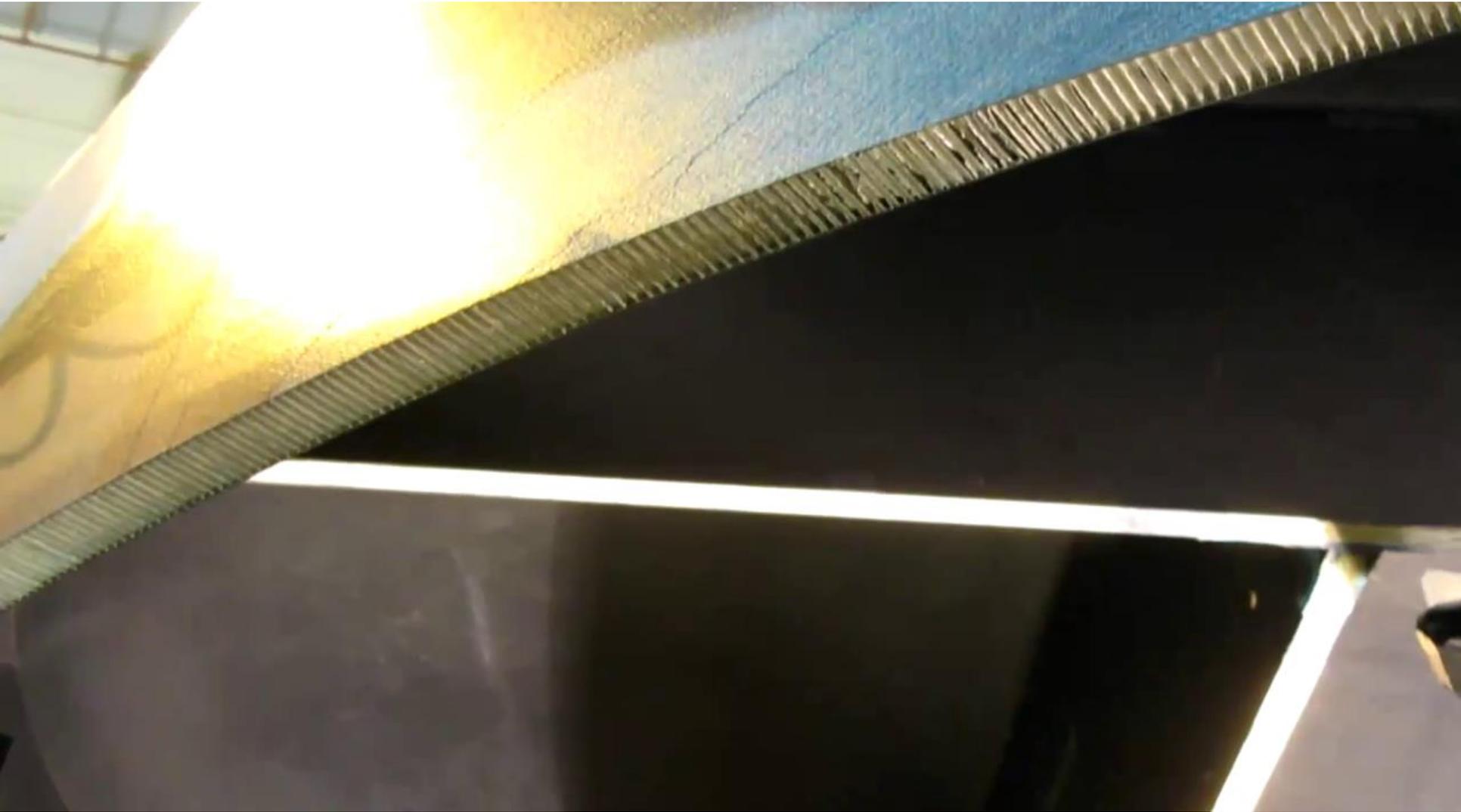
- Program launched in 2007
- Originally scheduled for entry into service in 2013
- Program delayed to 2014 due to several new technical challenges (Fuselage and empennage manufactured using an out-of-autoclave process (average elevation 2,000m, vacuum pressure -20%))
- Program suspended and 1,000 associated jobs lost January 2015

# Hand Lay-up Application



- NASA Composite Space Capsule
- Primary structure: Carbon epoxy composite (IM7/977-2) and aluminium honeycomb for sandwich structure
- Manufacturing method: hand lay-up, vacuum bagging and autoclave curing

# Hand Lay-up Application



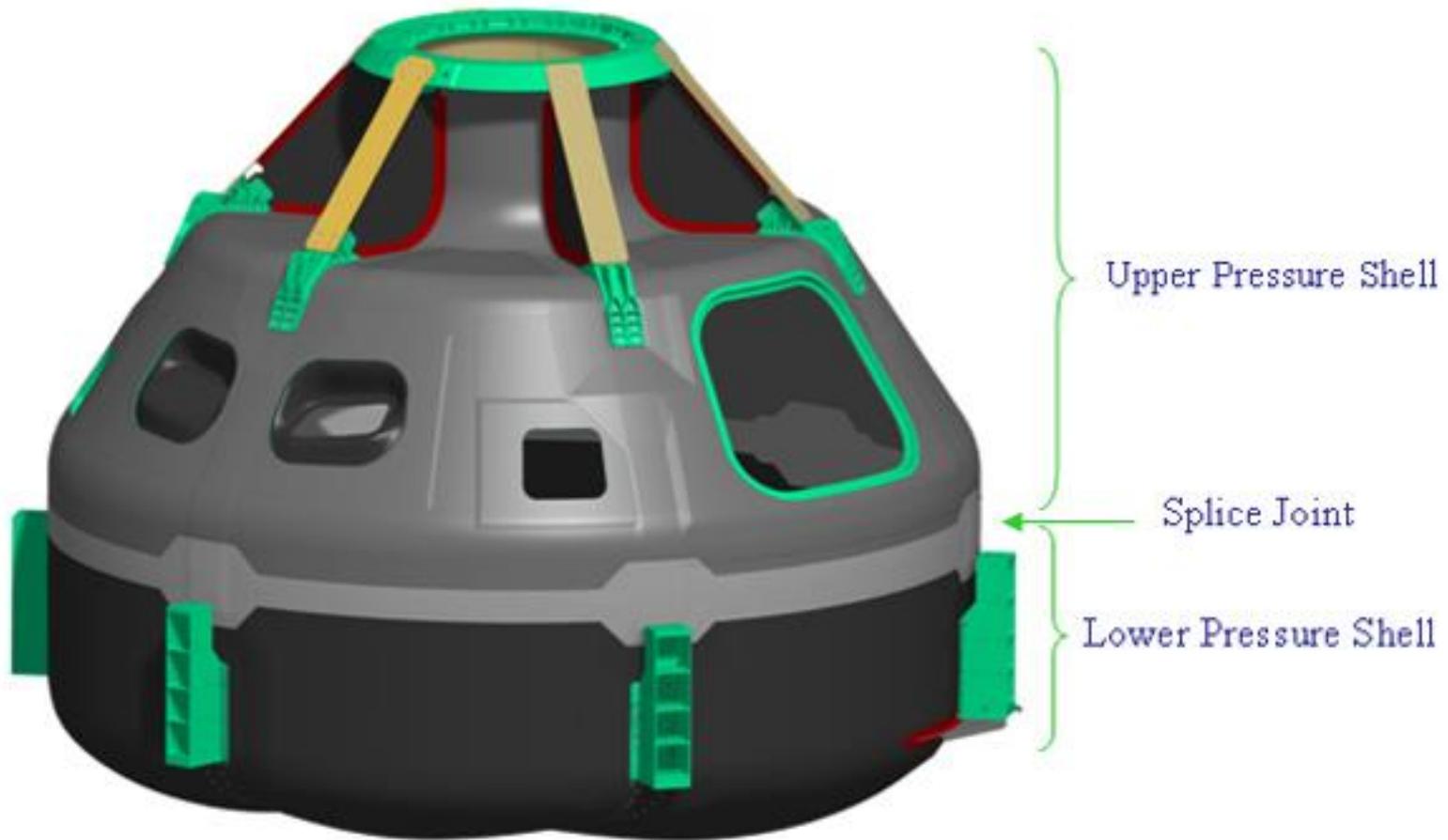
Composite sandwich structure: Aluminium honeycomb, composite facesheet

# Hand Lay-up Application



Aluminium honeycomb

# Hand Lay-up Application



Pressure Module = Upper Pressure Shell + Splice Joint + Lower Pressure Shell

# Hand Lay-up Application



Upper Pressure Shell cure tool

# Hand Lay-up Application



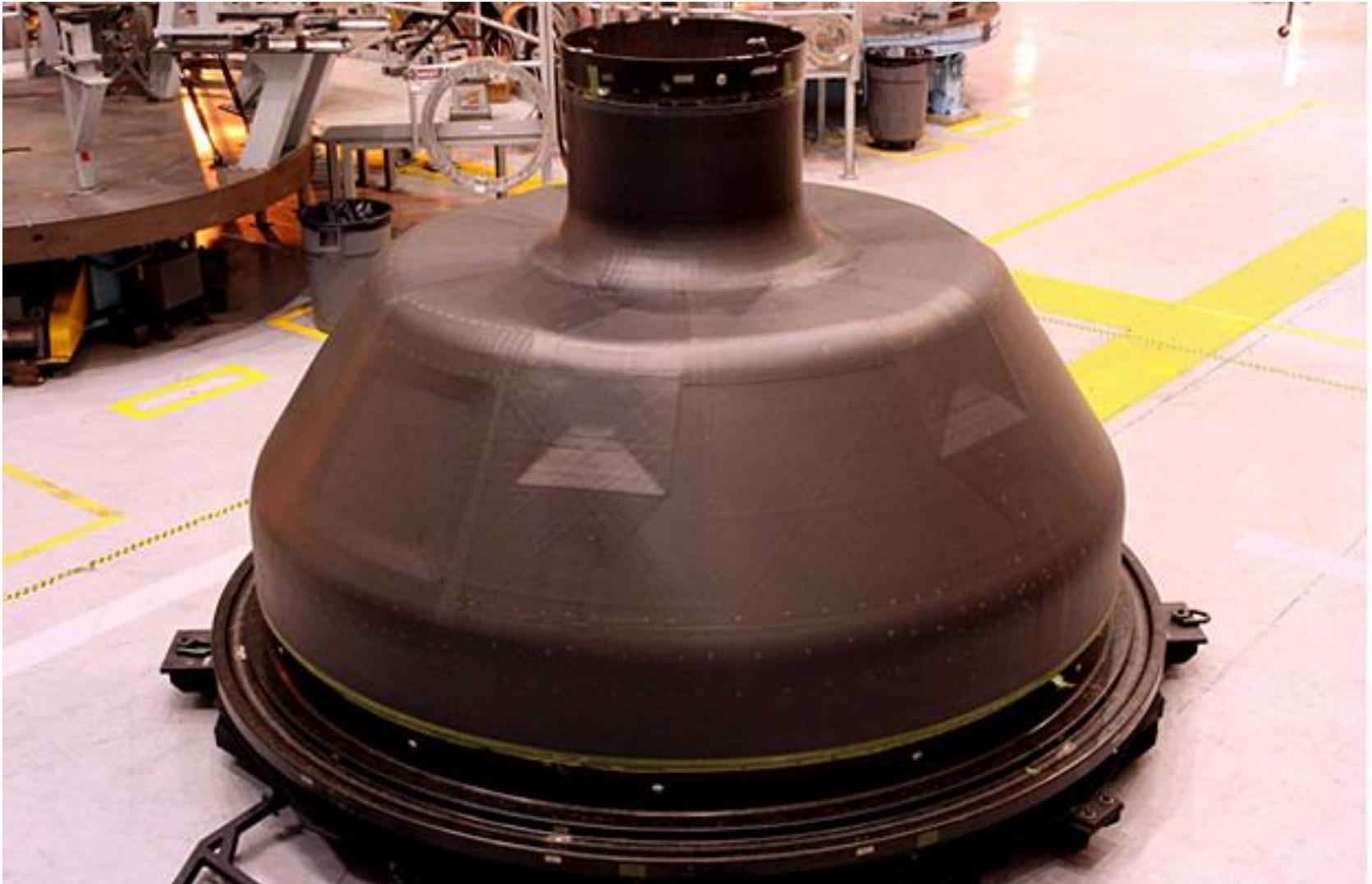
# Hand Lay-up Application



# Hand Lay-up Application



# Hand Lay-up Application



Cured Upper Pressure Shell Sandwich

# Hand Lay-up Application



# Hand Lay-up Application



Cured inner sandwich skin of Lower Pressure Shell

# Hand Lay-up Application



Upside Down Configuration within the Self Reacting Load Frame

# Success Stories: Composite Blades

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GE Composite Blades



The Museum of Modern  
Art of New York

[Hand lay-up example video](#)

# Liquid Composite Moulding

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- **Resin Transfer Moulding (RTM):** Mould is loaded with reinforcement and closed, resin is injected into mould and then cured. Mould is often put under vacuum, VARTM (vacuum assisted resin transfer moulding).
- **Compression moulding:** A dry mat of reinforcing material is preformed and placed into an open mould, Resin is added, and the mould halves are pressed together and heated.
- **Infusion:** An open mould process that uses vacuum pressure to drive resin into the composite laminate.



# RTM Application

- The Lamborghini Aventador LP700-4 is a two-door, two-seater sports car.
- The carbon fiber composite monocoque is manufactured using an RTM process.



RTM process



carbon fibre composite monocoque

# Compression Moulding Application

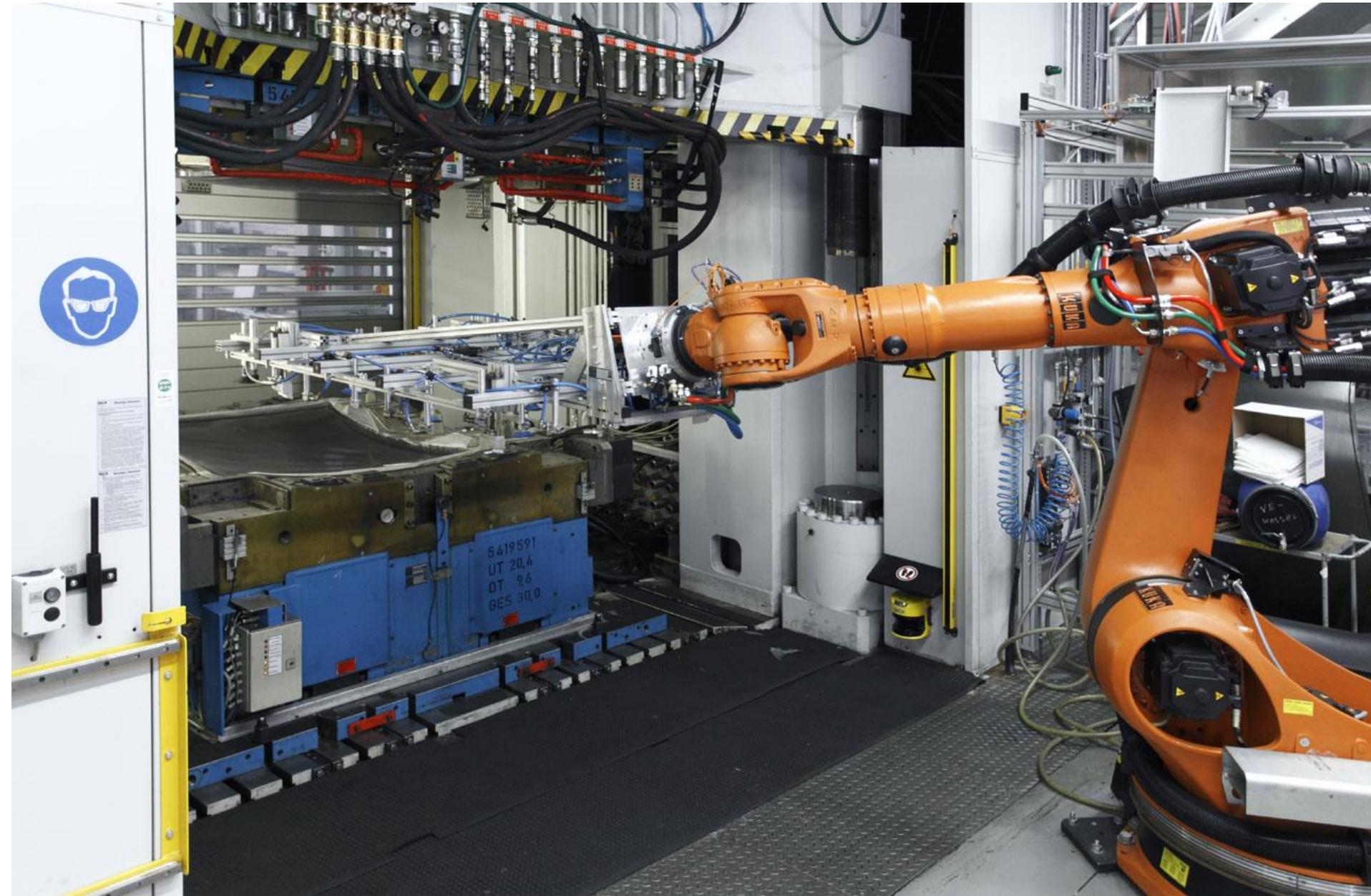
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- BMW i3, is an urban electric car under by BMW
- The i3 is the first mass production car to have most of its internal structure and body being made of carbon-fiber reinforced plastic (CFRP)





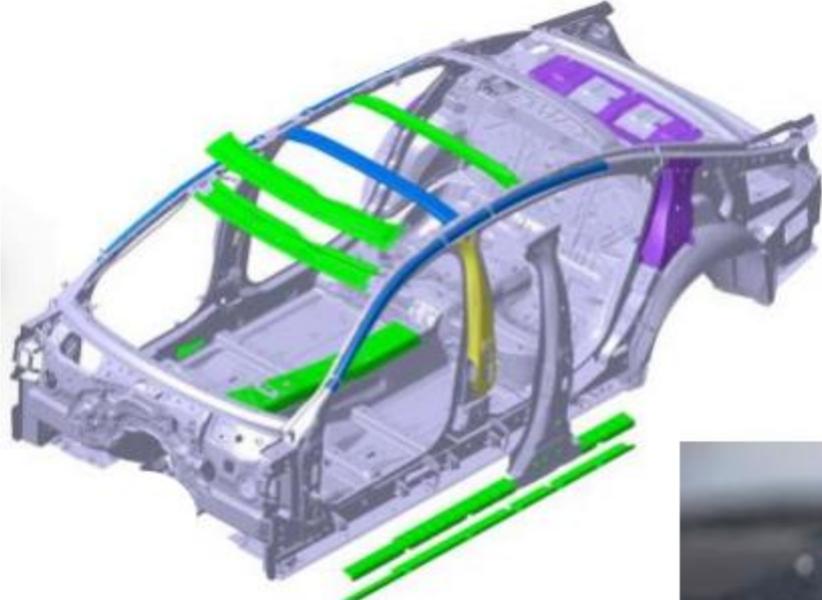
# Compression Moulding Application



# Compression Moulding Application



# Processes: Wet Compression Molding: BMW 7 Series



CFRP Wet Compression Molding

CFRP-Steel Hybrid

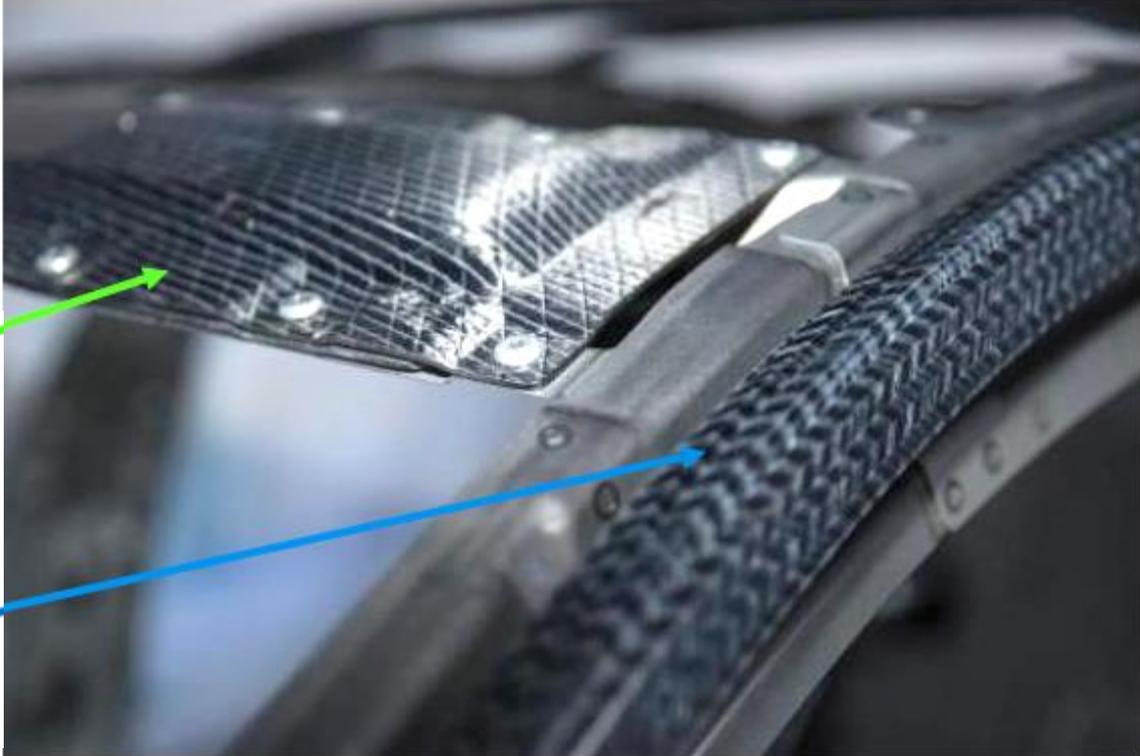
CFRP Resin Transfer Molding

CFRP Sheet Molding Compound

[example video](#)

CFRP Wet Compression Molding

CFRP Resin Transfer Molding



# Infusion Process

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25 Challenger- Maritime (9 m), Manufacturing Process

# Infusion Process

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Inspection and maintenance of the mold

# Infusion Process

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Applying gel coat for color and protection

# Infusion Process

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Placing glass fabric in the mould

# Infusion Process

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A nylon bag is placed over the entire hull mould and sealed around it

# Infusion Process

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Resin is drawn into the mould by using vacuum lines

# Infusion Process

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# Infusion Process

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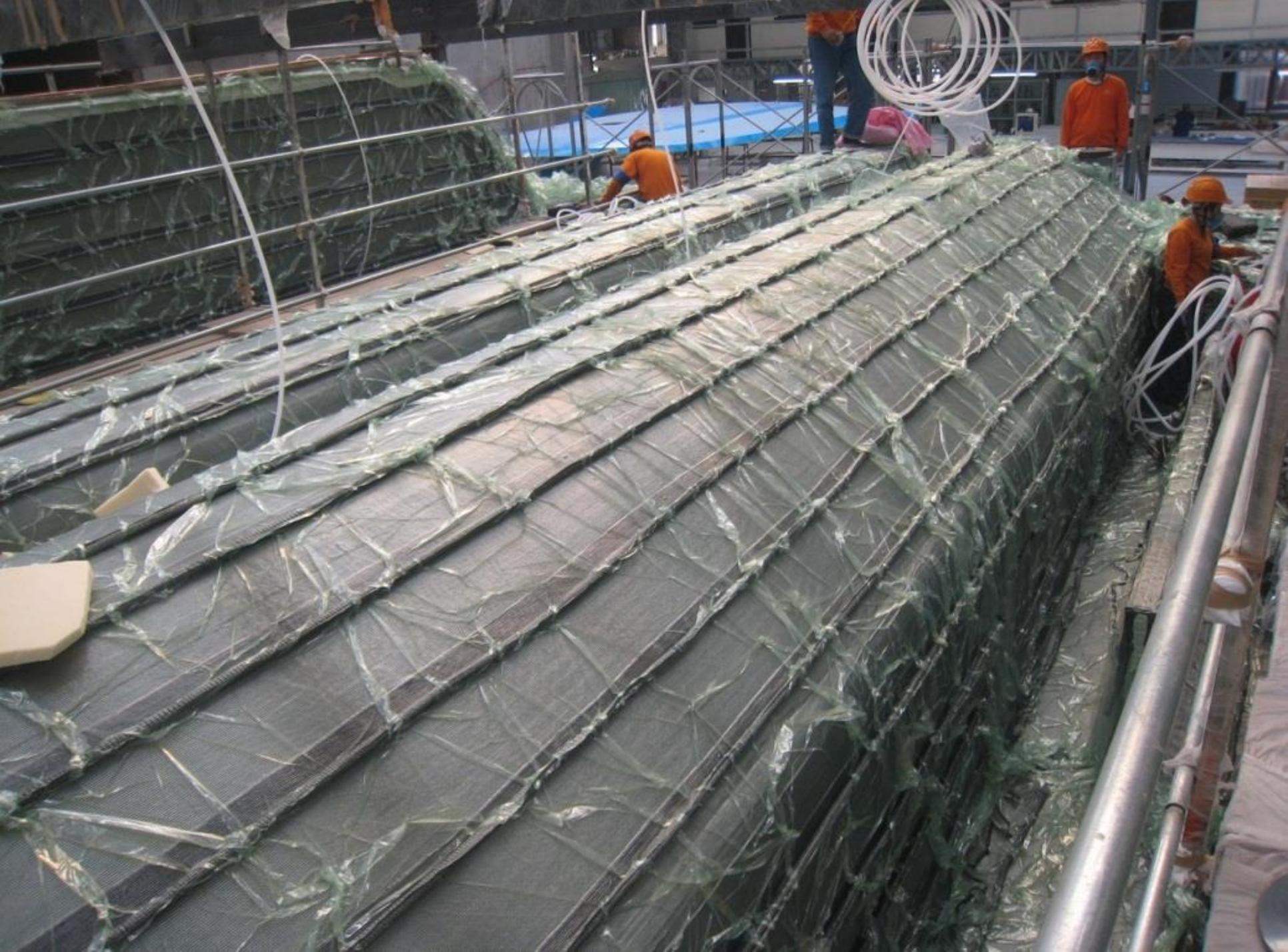
Resin is then cured

# Infusion Applications

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- Horizon PC58 manufactured by the Horizon Company as a new model of luxury power catamaran (18 m)
- Fiber glass epoxy and Divinycell foam for the sandwich layers







# Braiding



# Braiding



- Radial braider with 144 carriers with a robotic arm
- The speed of the machine can go up to 200 r.p.m.

# Braiding Applications

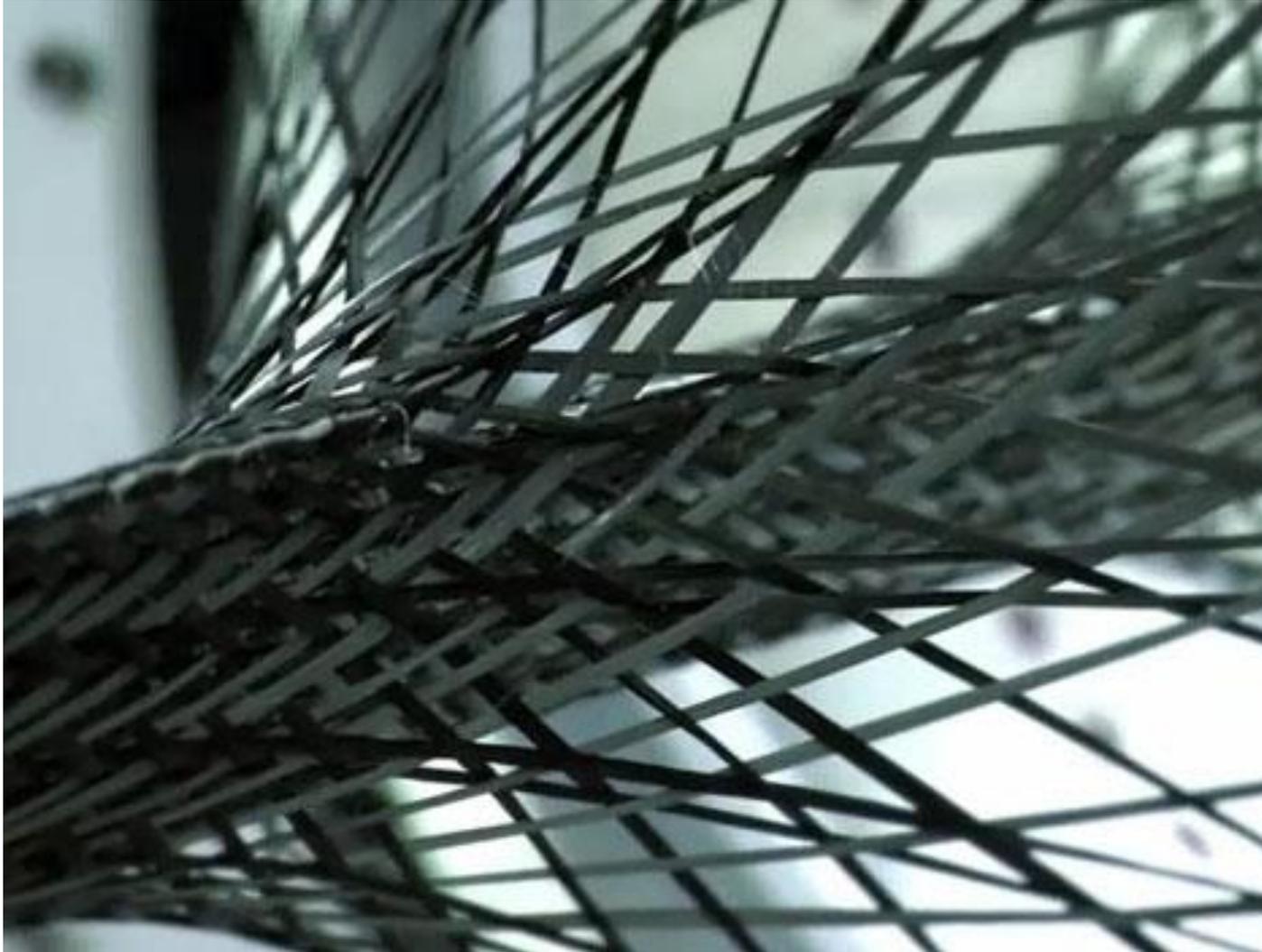
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Braiding is used to produce CFRP parts for Lexus LFA using a special 3D carbon-braiding machine .



# Braiding Applications

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# Automated Fiber Placement (AFP)

- AFP uses an automated machine to place fiber onto a tool
- Two main styles:
  - Head mounted on a robotic arm
  - Head mounted on gantry system



Boeing 777x Wing

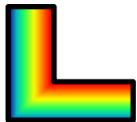


Boeing 787 Fuselage

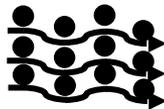
# Process Simulation

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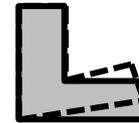
- For proper control of temperature and pressure history during processing, we can take advantage of process simulation.
- Currently, one of the main trends in process simulation is to break down the complexity of the system into several simpler sub-models:
  - Thermo-chemical analysis
  - Resin flow and compaction analysis
  - Stress and deformation analysis



Thermo-Chemical



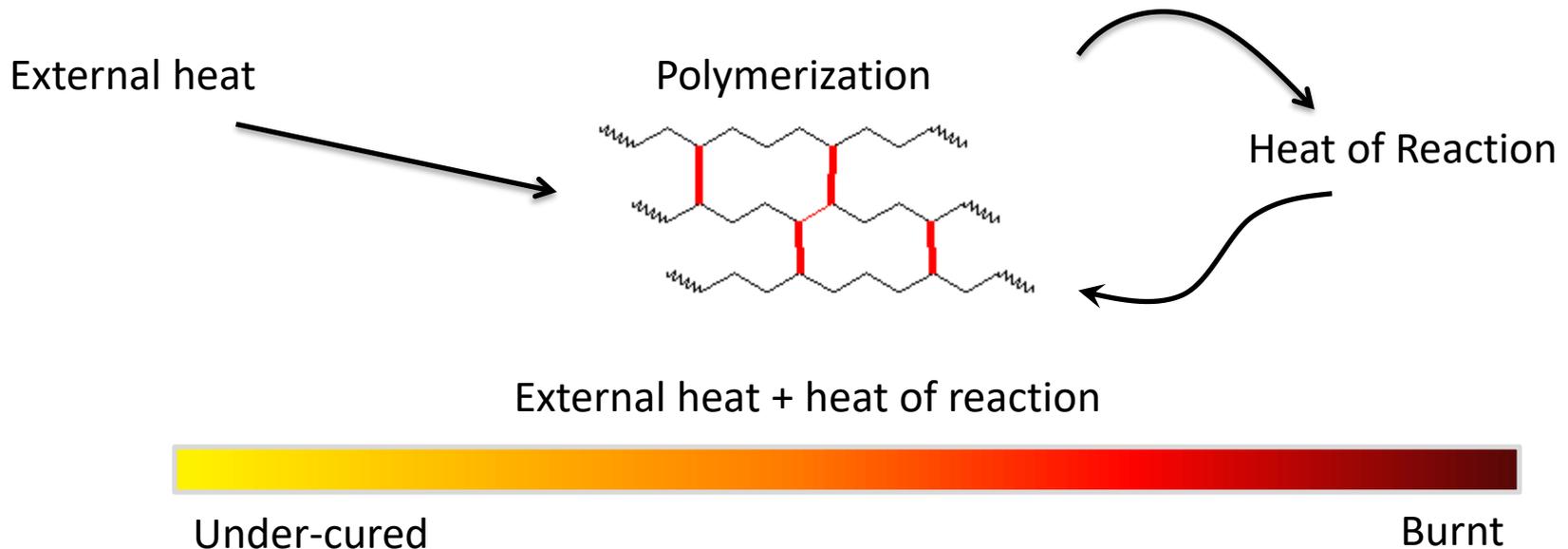
Flow-Compaction



Stress-Deformation

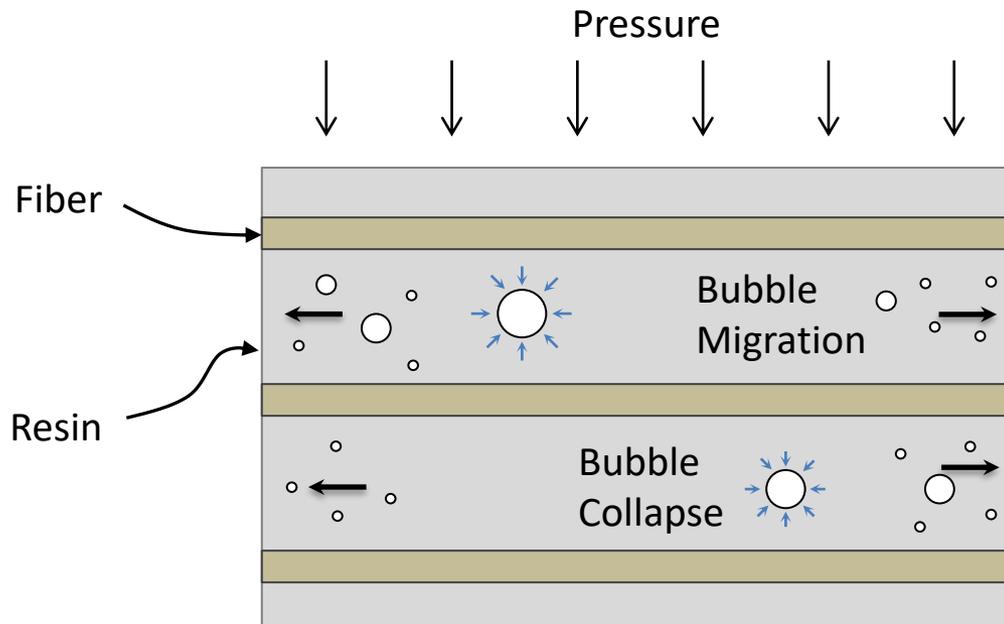
# Introduction: Thermo-chemical reaction

- *Thermo-chemical reaction of the resin:*
  - Polymerization of thermoset resins is an exothermic reaction and heat is generated during the curing process. Usually, the heat of reaction is not enough to fully cure the resin and external heat is needed to advance the curing of resin. By providing enough external heat during the fabrication process, a desirable degree of cure (DOC) throughout the part is achieved. However, too much heat (sum of external and internal heats) damages the resin by burning it.



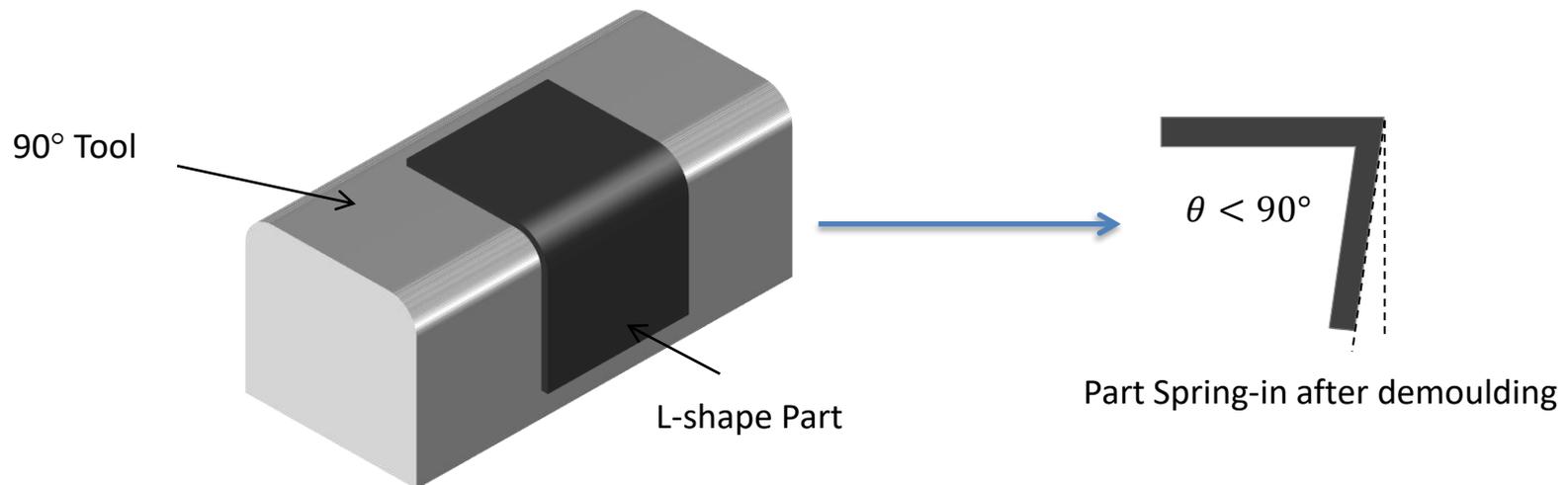
# Introduction: Flow and Compaction

- *Resin flow and fiber compaction:*
  - At the beginning of the fabrication process and while the resin is still in its liquid form (before gelation), by increasing the temperature, resin viscosity drops significantly. This drop in viscosity combined with the applied pressure on the part, helps to remove trapped air bubbles in the resin through bubble migration or bubble collapse. Although heat is needed to reduce the viscosity of resin, too much heat might result in early gelation of the part before bubbles get a chance to migrate or collapse.



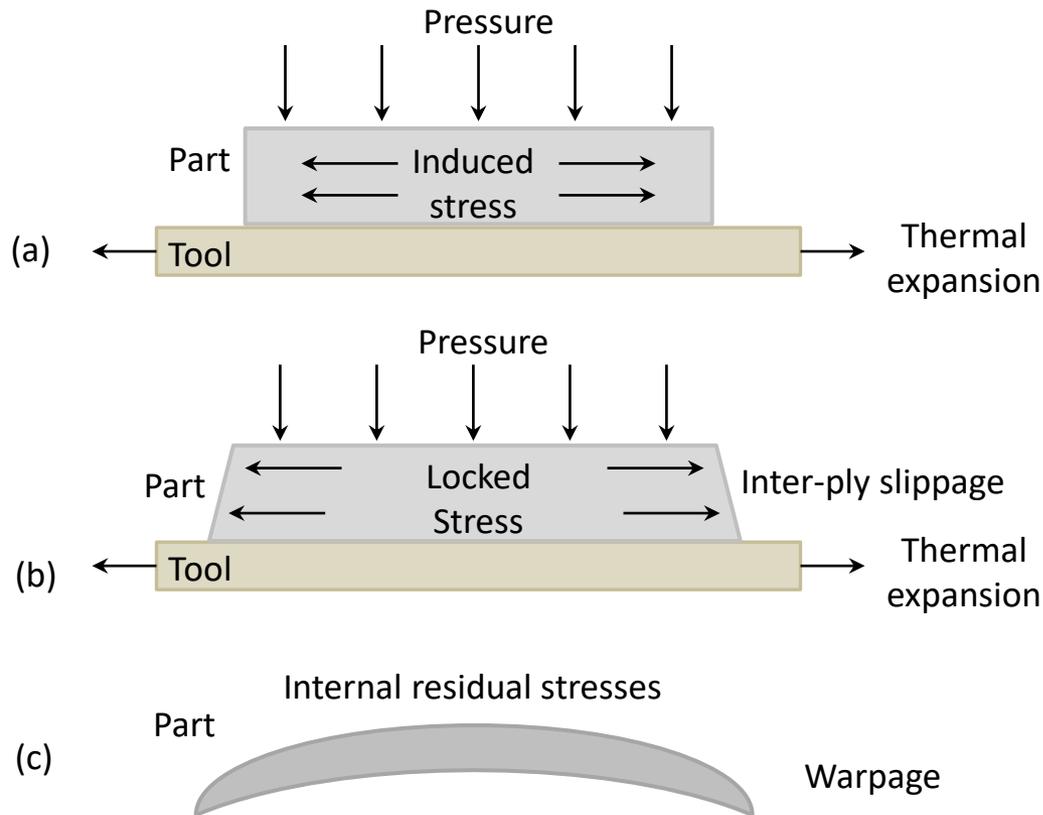
# Introduction: Residual Stress and Dimensional Changes

- *Residual stress formation and dimensional changes:*
  - Composite materials have high through-thickness CTE (Coefficient of Thermal Expansion) and low in-plane CTE along the fiber direction. Tool CTE is usually different than the part CTE as well. Aside from mismatch of CTEs, during the polymerisation process, resin shrinks while fibers do not shrink. These mismatch of free strains (sum of thermal strains and resin shrinkage) between fibers, resin and tool, results in the formation of residual stresses during the fabrication process. This leads to dimensional changes in the composite part including warpage of flat parts and spring-in of curved parts.



# Introduction: Residual Stress and Dimensional Changes

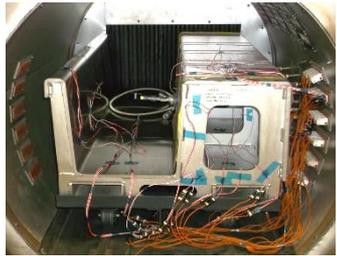
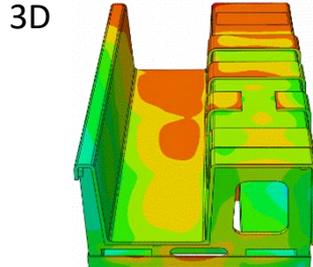
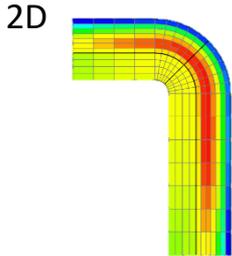
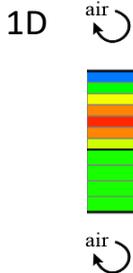
- For example, warpage formation in flat composite parts is explained here. At the beginning of the fabrication process, as the part and tool are heated together, tool expands while the part doesn't expand as much as the tool due to the low CTE of the fibers (a). This creates a through-thickness residual stress distribution in the part due to ply slippage while the resin is still soft (b). After demoulding, this residual stress creates warpage in the part (c).



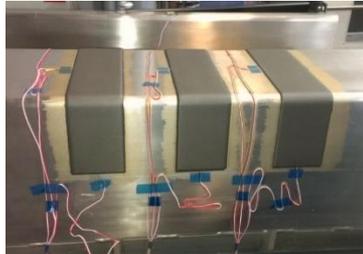
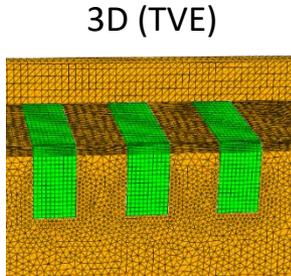
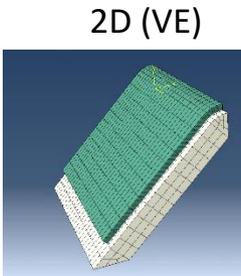
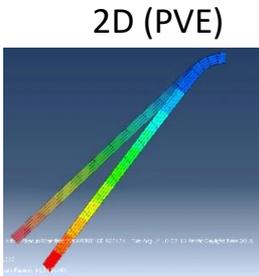
Process Simulation

Experiment

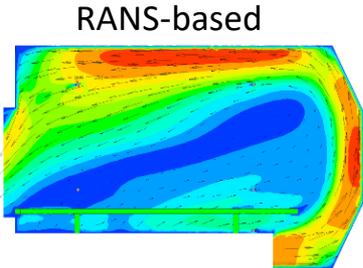
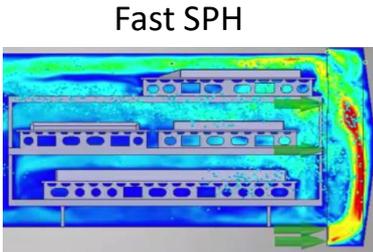
Thermo-Chemical Analysis



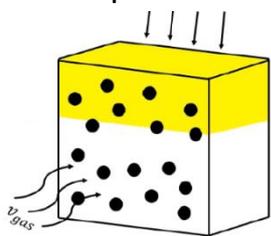
Stress-deformation Analysis



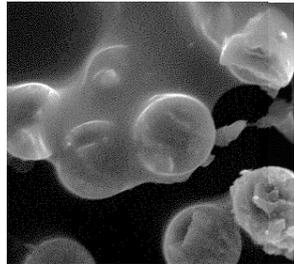
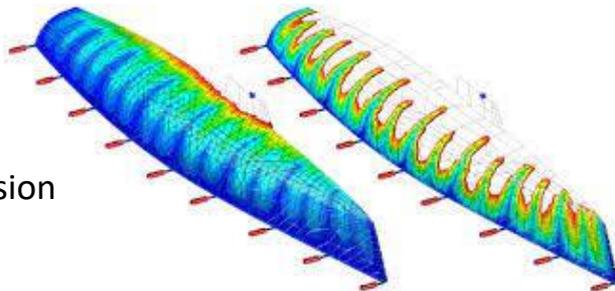
Flow and Compaction Analysis



Resin/Gas coupled flow

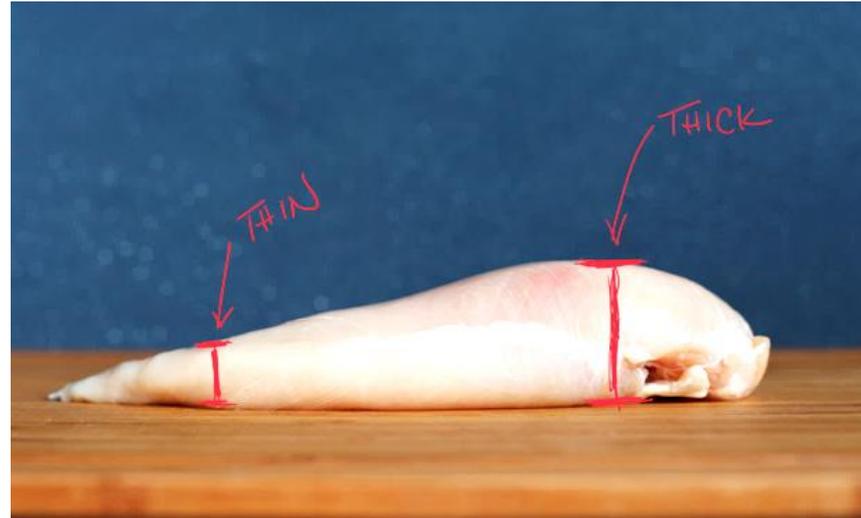


3D infusion



# Thermo-chemical Analysis: A Cooking Analogy

- While cooking chicken, uneven thicknesses create problems.
- Thin sections are cooked first and become dry while thick sections may be partially cooked by the end
- We try to cook the chicken to an even temperature of 165 °F

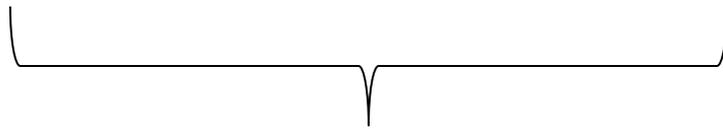


< 160 °F	165 ± 5 °F	> 170 °F
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Uncooked

Cooked

Dry/Burnt



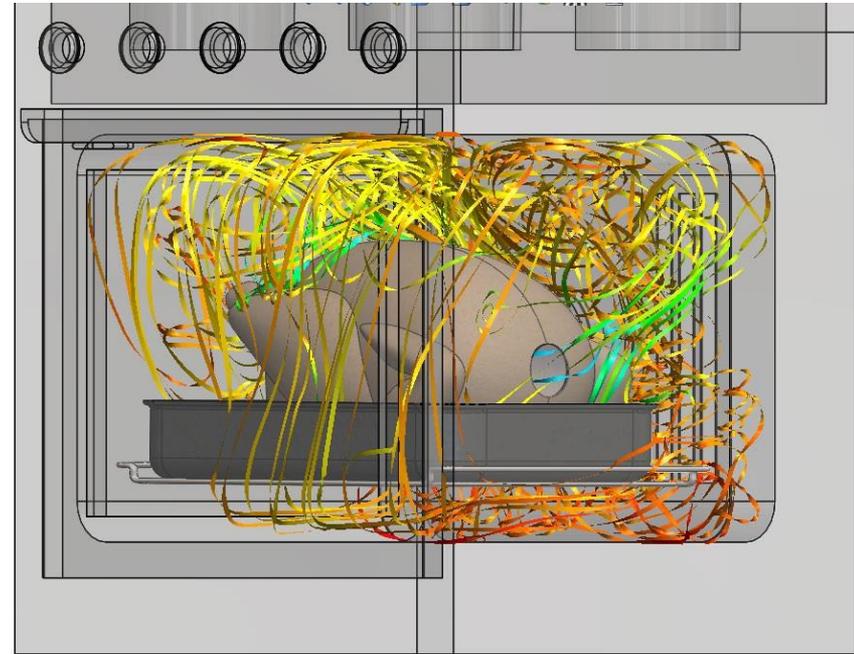
This is similar to the specifications in composites manufacturing



# Thermo-chemical Analysis: A Cooking Analogy

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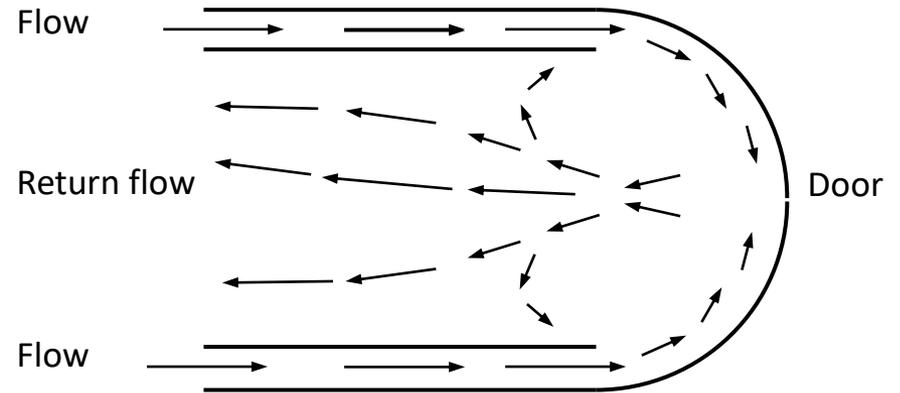
- For a more accurate temperature control, it is better to slowly cook the chicken in a convection oven:
  - Airflow increases the rate of heat transfer and creates a uniform temperature distribution
  - Slow heating rates and long cooking times ensure uniform cooking
- This is similar to thermal management in composites manufacturing
- The main difference is the heat generation in the composite part due to the exothermic reaction
- Unlike cooking, due to cost consideration, engineers try to avoid slow heating rates and long cooking times



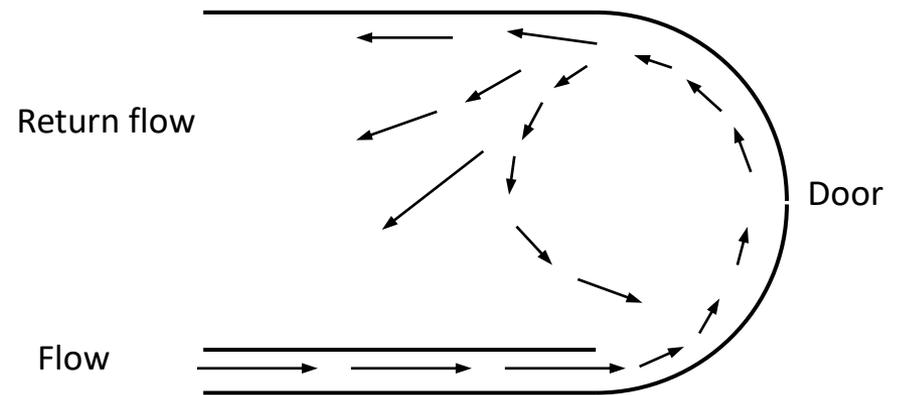
# Equipment: Autoclave - Airflow



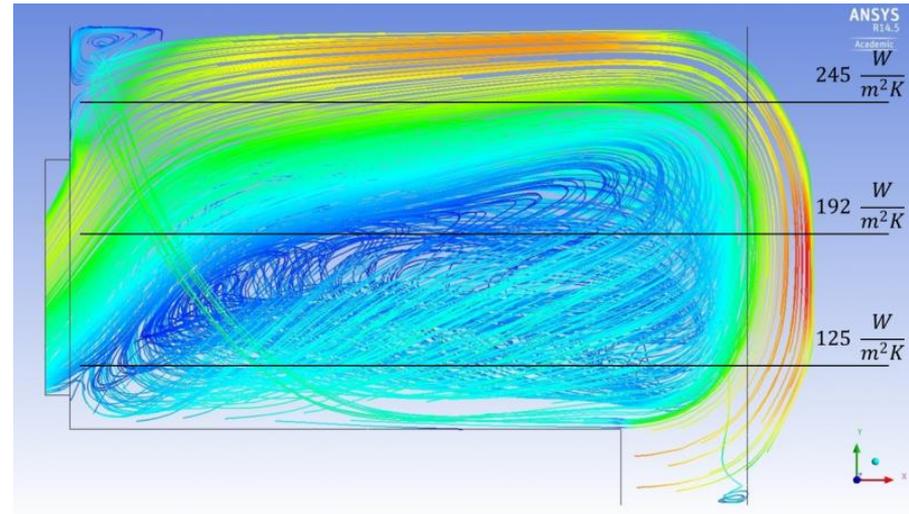
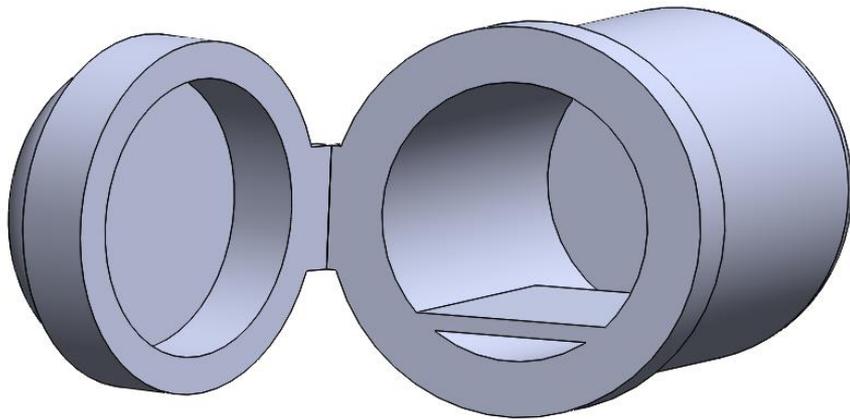
**Annulus air duct**



**Floor air duct**



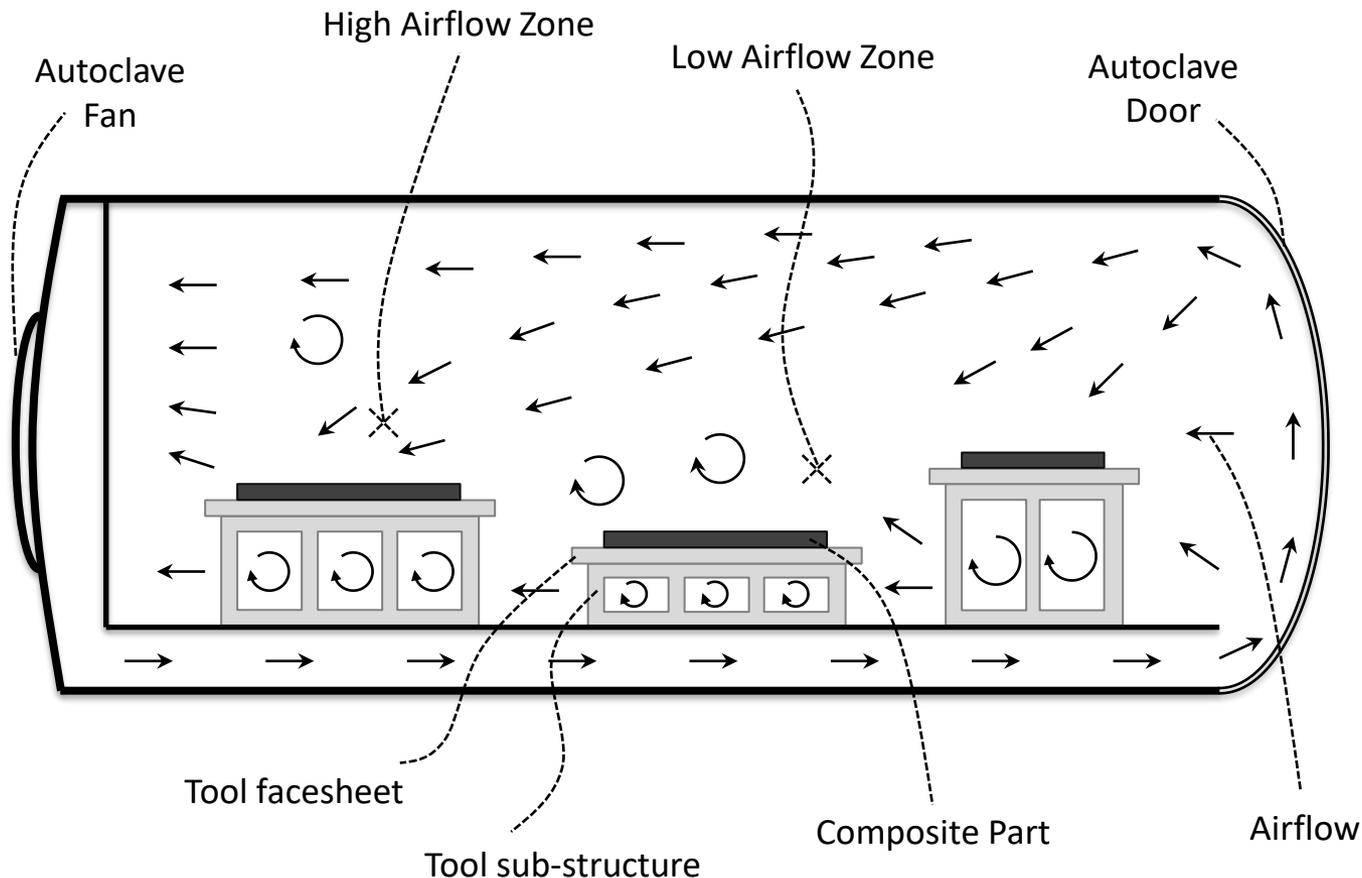
# Equipment: Autoclave - Airflow



Airflow

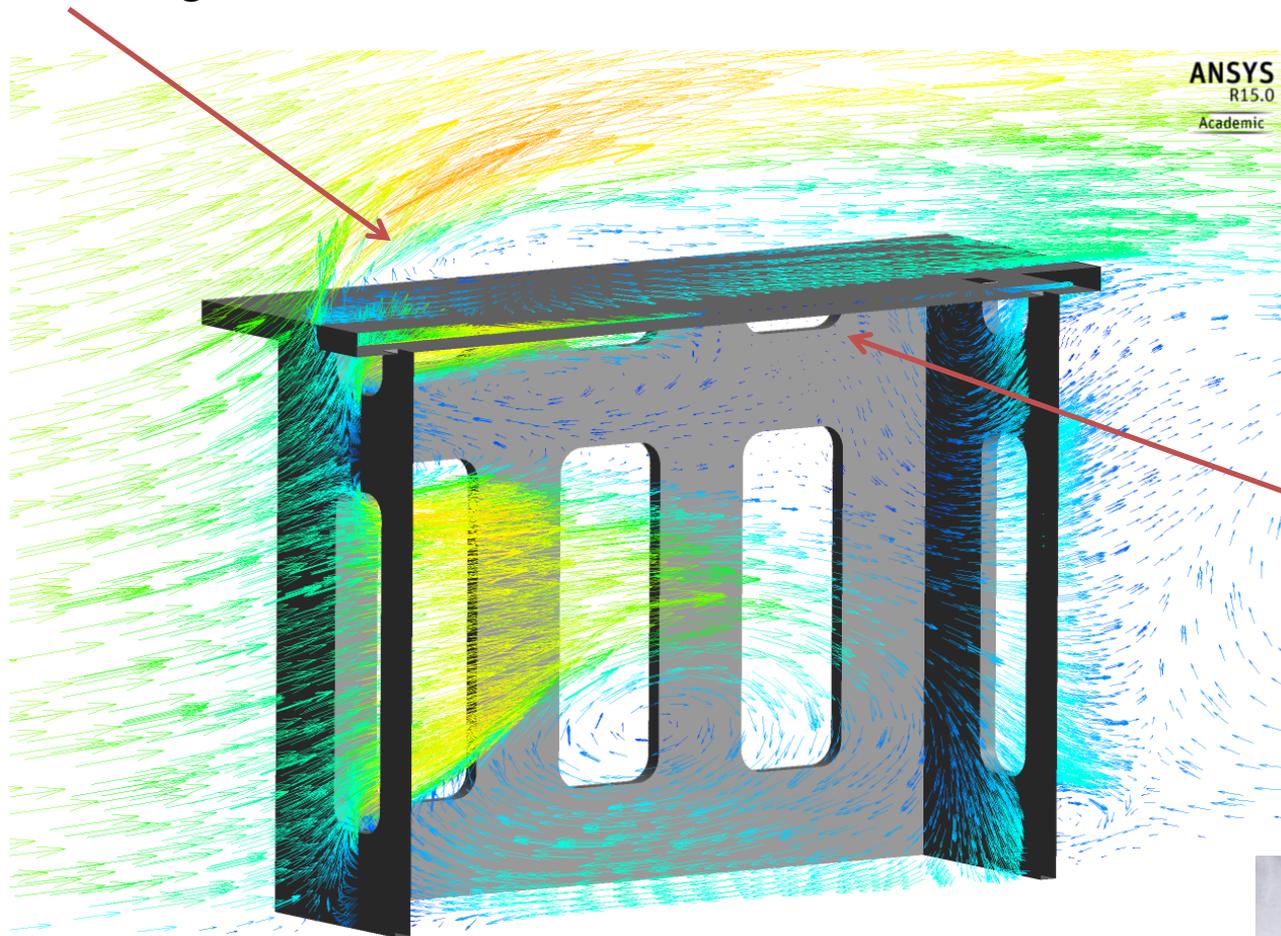
# Convective Curing of Thermoset Composites

- In convective curing (e.g. autoclave or oven curing) part and tool are heated together using convection to cure the composite part.
- Following figure, for example, schematically shows the cross-sectional view of a typical production autoclave which is loaded with three tools and parts.



# Tool - Thermal Management: Effects of Sub-structure

Good airflow: High HTC



Bad airflow:  
Low HTC

CFD analysis on half of a steel tool



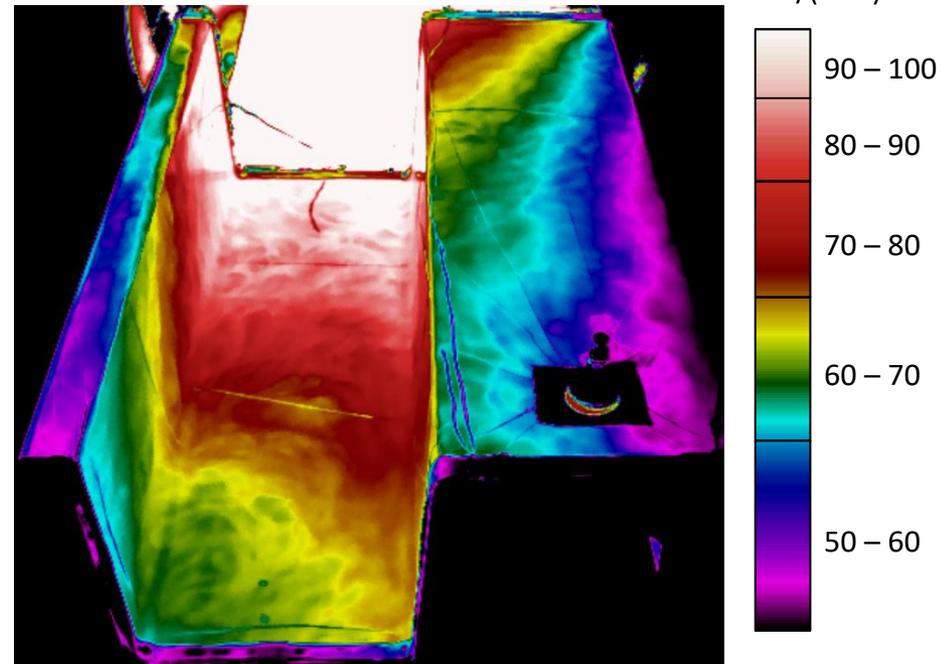
# IR Thermography To Evaluate BCs

- Tools were vacuumed bagged and then spray painted with a matte paint.
- They were heated in an autoclave before the door was opened quickly and an IR image was taken. Using temperature distribution and FE, heat transfer coefficients were back calculated:

**Tool Covered with Painted Vacuum Bag**

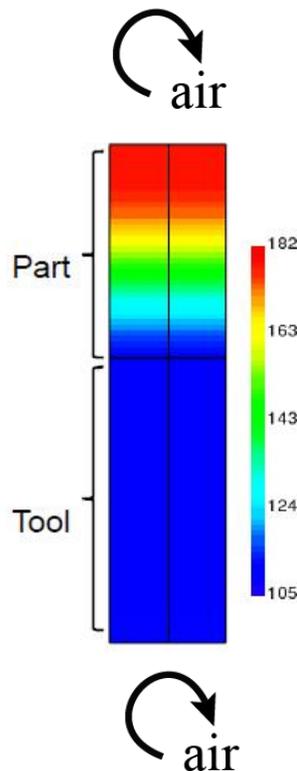


**Distribution of Heat Transfer Coefficients**



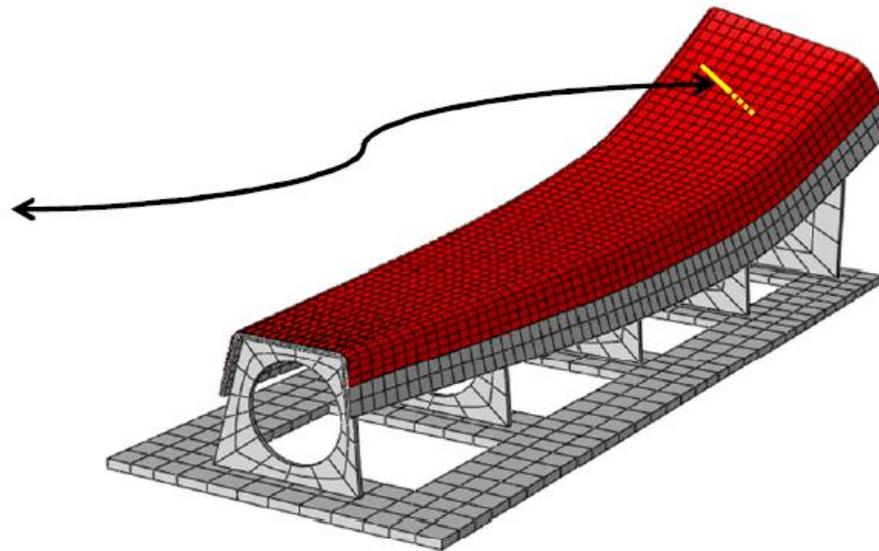
# Thermo-Chemical Analysis of Composites

- Approximately, we can simplify the complex 3D thermo-chemical problem into a 1D problem.
- This is because through-thickness heat transfer is the dominant mechanism away from the edges and sub-structures.



1D approximation

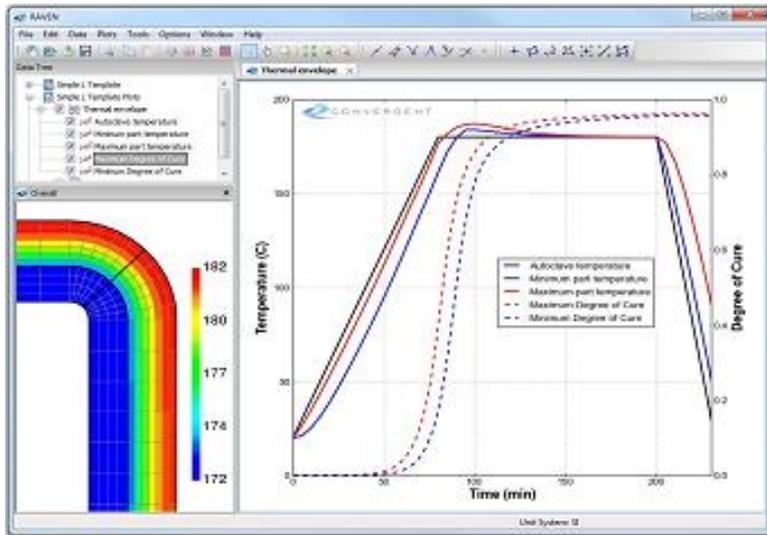
$$\frac{\partial}{\partial t}(\rho C_P T) = \frac{\partial}{\partial x} \left( k_{xx} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{yy} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_{zz} \frac{\partial T}{\partial z} \right) + \dot{q}$$



3D thermo-chemical  
problem

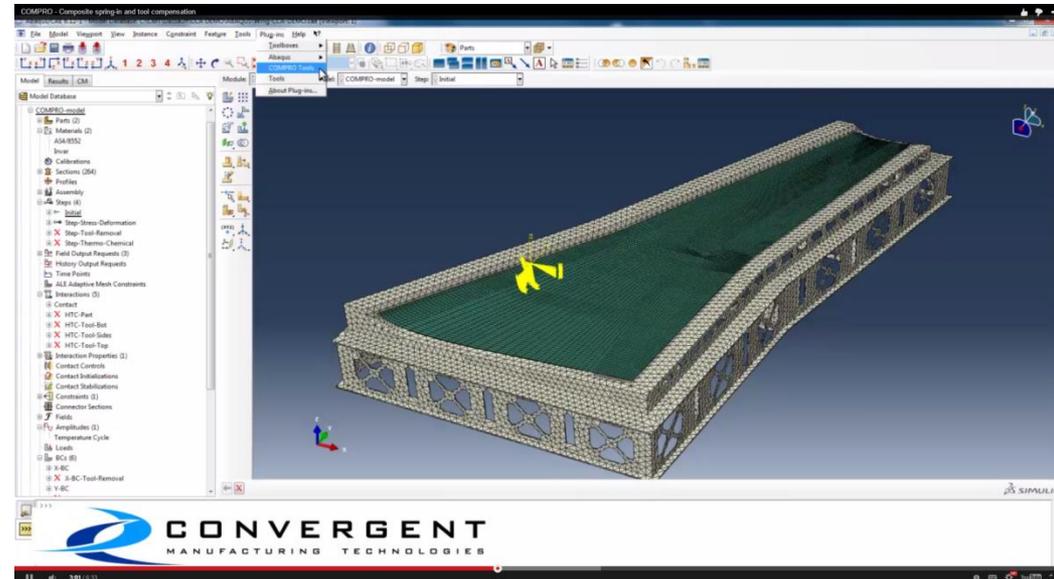
# Commercial Simulation Tools

- Simulation tools are available for thermal management in composites manufacturing.



RAVEN

0D, 1D, Limited 2D



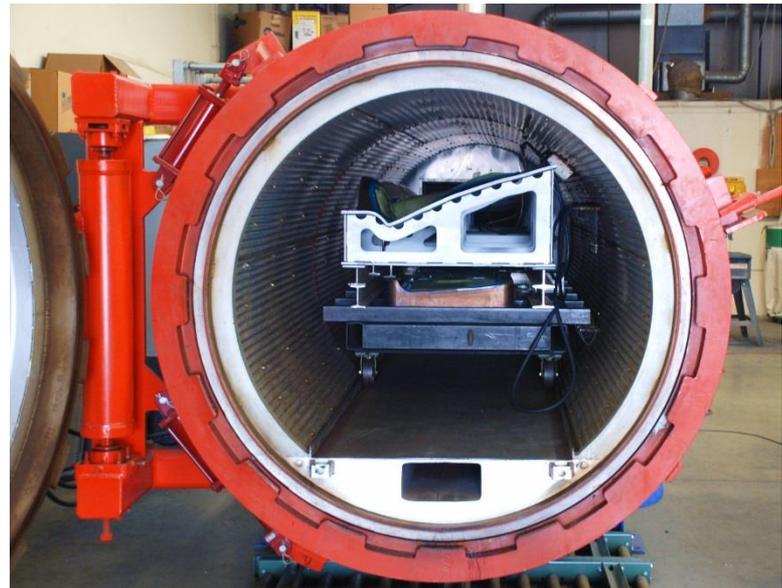
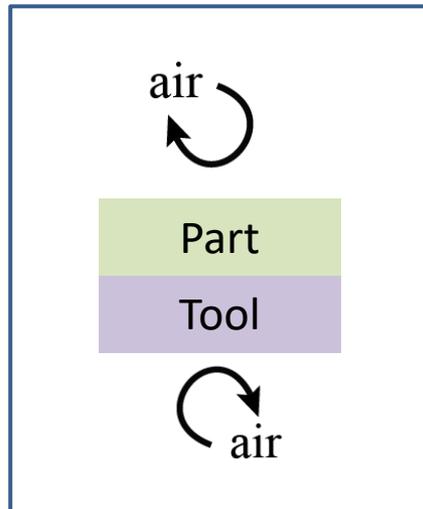
COMPRO 3D

# 1D System Definition: Autoclave/Oven Curing

- Heat transfer to the composite part:
  - Convective heat transfer from top of the part and bottom of the tool
  - Conductive heat transfer through the tool
- During the heat-up step, the tool transfers the heat to the composite part. During curing, tool acts as a heat sink for the exotherm energy.

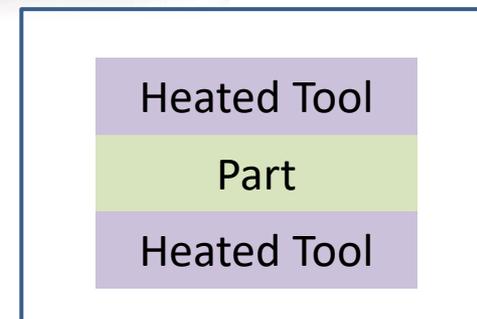
Applied Temperature and Pressure Cycle

System definition



# Heated mould: Closed-mould Process

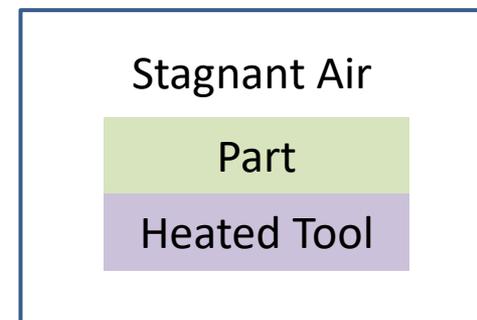
- Heat transfer to the composite part:
  - Conductive heat transfer through the tool
- Manufacturing processes such as Compression Moulding or RTM use this curing process.



System definition

# Heated Mould: 1-Sided mould

- Heat transfer to the composite part:
  - Mainly conductive heat transfer through the tool side
- Manufacturing processes such as infusion use this curing process.



System definition

# Room Temperature Curing

- Room temperature curing has very low heat transfer through the boundary (stagnant air). Consequently, there is low control over the curing process with wide distribution of temperature.

Stagnant Air

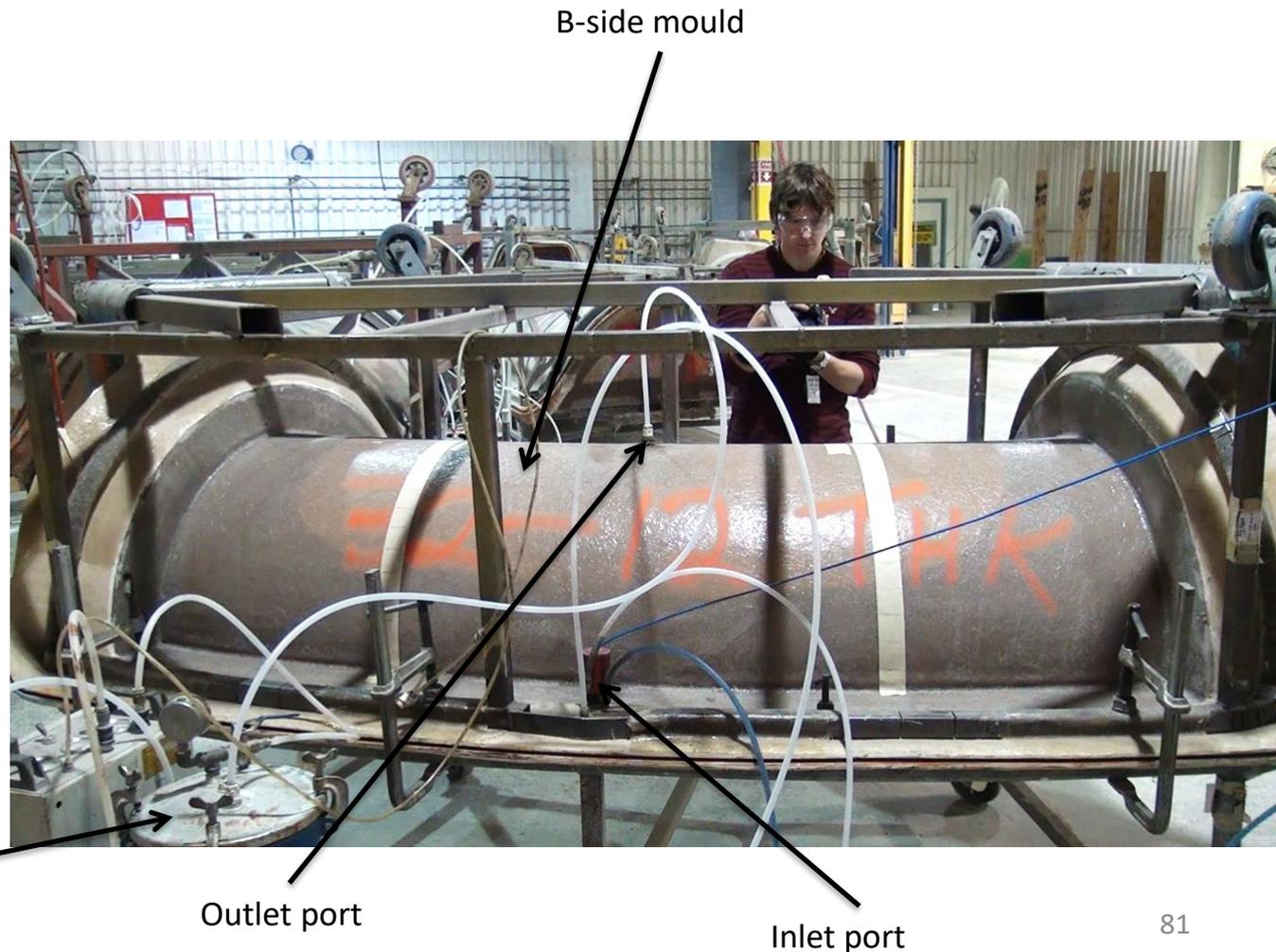
Tool

Part

Tool

Stagnant Air

System definition



B-side mould

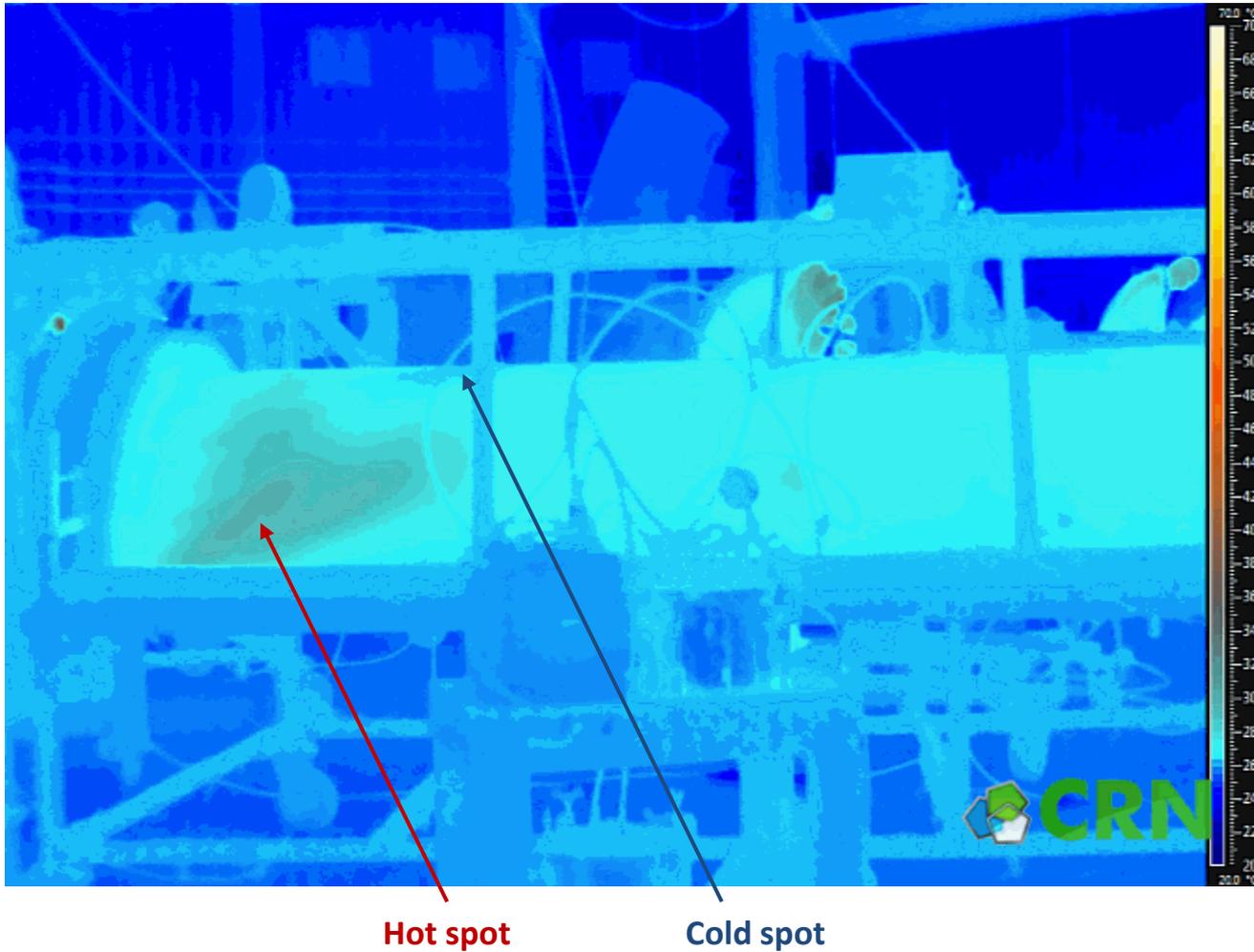
Resin catch  
pot

Outlet port

Inlet port

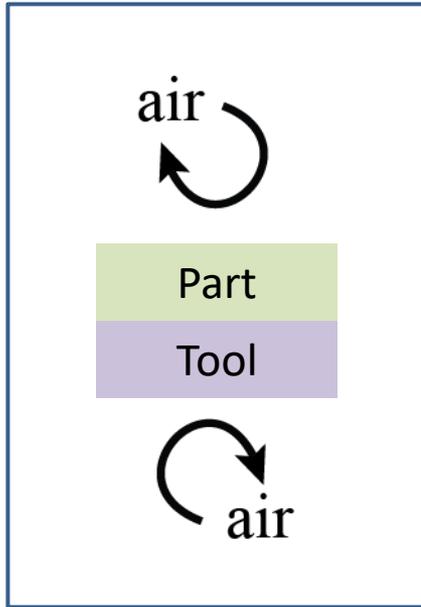
# Room Temperature Curing

Wide PROCESSING TEMPERATURE DISTRIBUTION



# Summary: Different Curing Processes

## Autoclave/Oven



Applied Temperature and Pressure Cycle

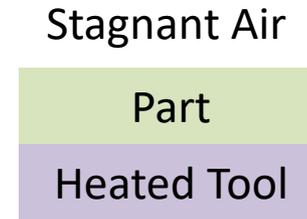
- Aerospace
- Space
- Defense
- High-end Automotive
- Sport

## Hot Press/ Heated Mould



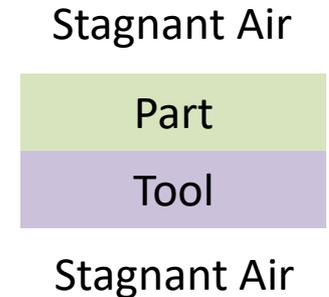
- Automotive
- Aerospace
- Sport

## 1-Side Heated Mould



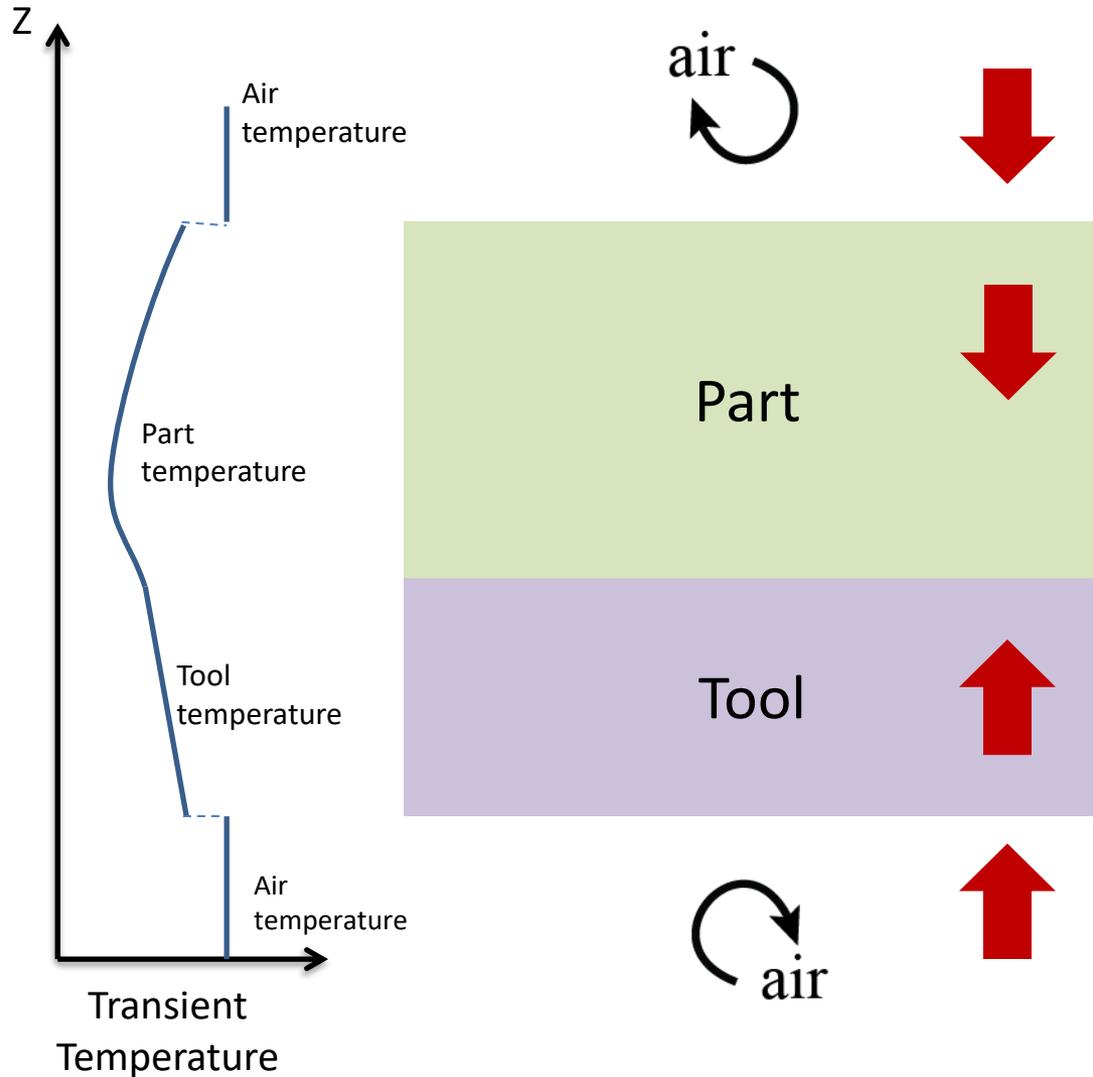
- Wind Energy

## Room Temperature Cure



- Marine
- Sport
- Construction
- Industrial

# 1D Thermo-Chemical Analysis: Heat-up Step



Rate of heat transfer by convection

$$\dot{Q} = HTC \times A \times \Delta T$$

Energy balance:

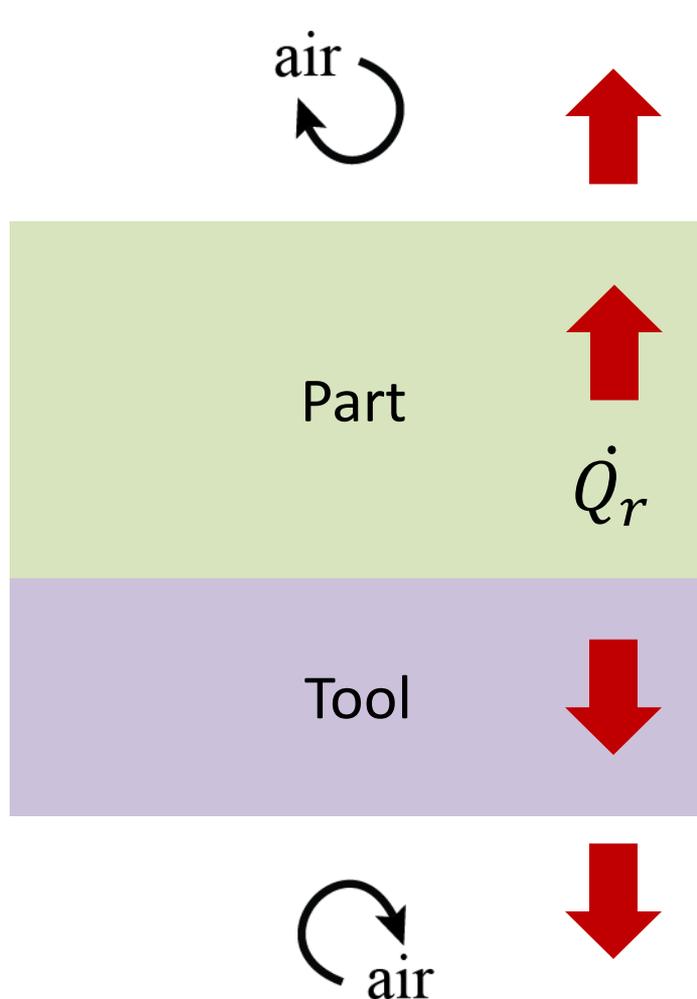
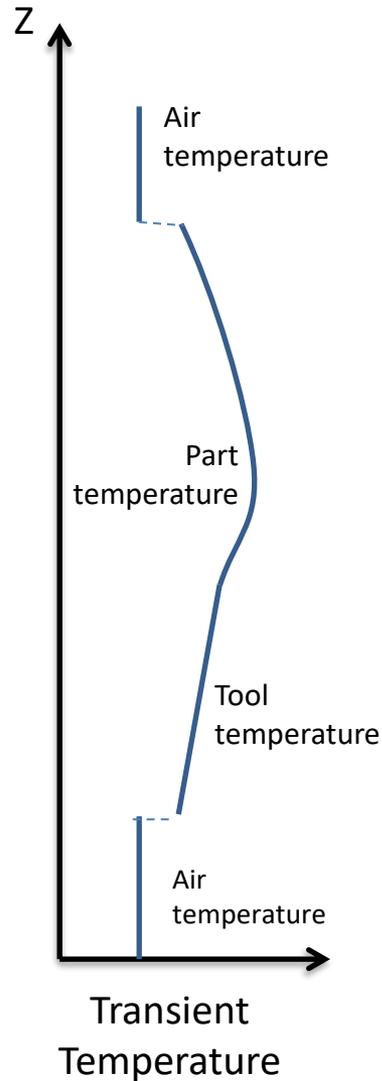
$$\frac{\partial}{\partial t} (\rho C_p T) = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right)$$

rate of heat increase = rate of heat addition by conduction

Rate of heat transfer by convection

$$\dot{Q} = HTC \times A \times \Delta T$$

# 1D Thermo-Chemical Analysis: Polymerization Step



Rate of heat transfer by convection

$$\dot{Q} = HTC \times A \times \Delta T$$

Energy balance:

$$\frac{\partial}{\partial t} (\rho C_p T) = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{Q}_r$$

rate of heat increase = rate of heat addition by conduction + rate of resin heat generation

Rate of heat transfer by convection

$$\dot{Q} = HTC \times A \times \Delta T$$

# Cure Cycles

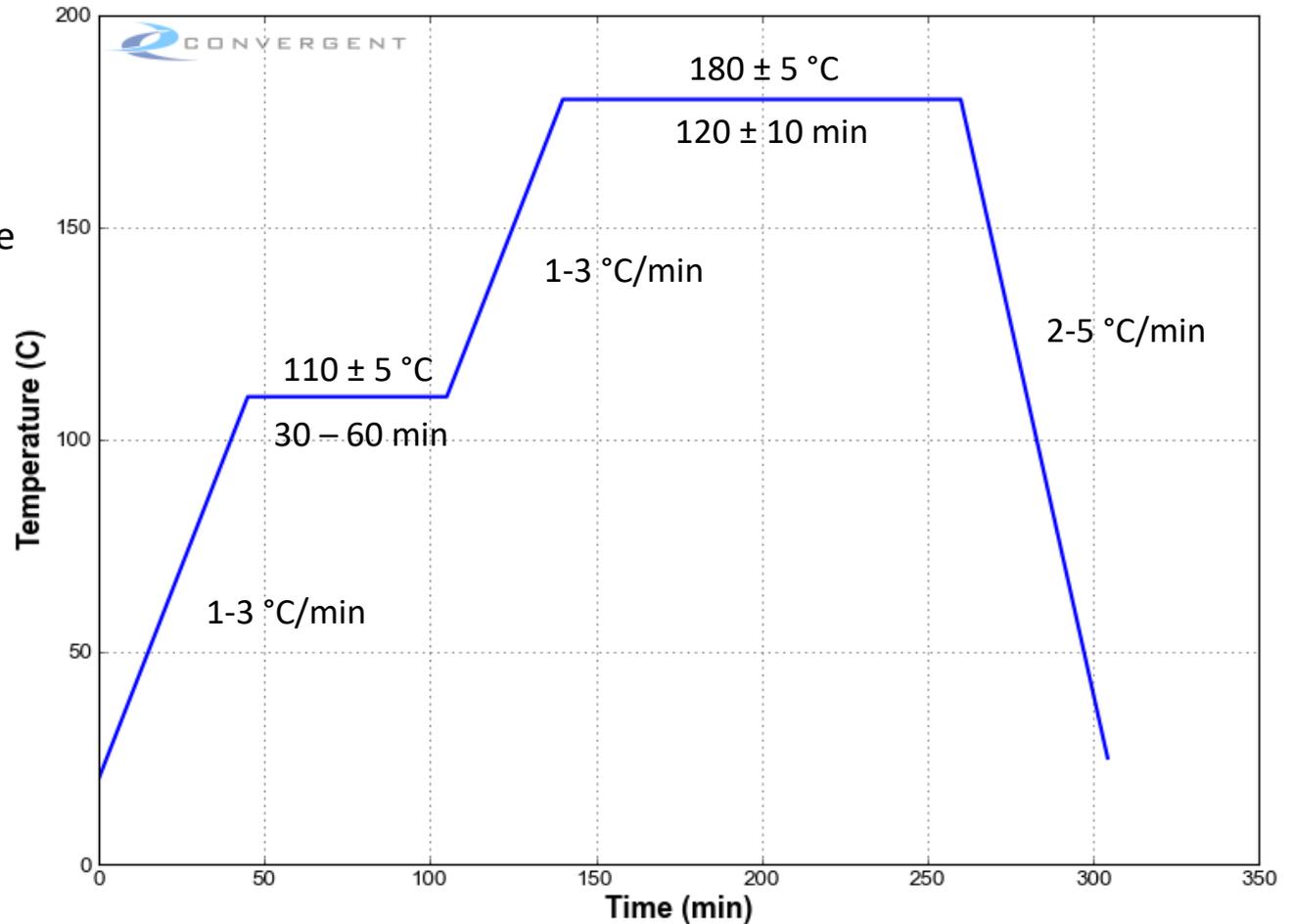
- Resin manufacturers usually recommend cure cycles (i.e. recipes).
- Here is a Manufacturer's recommended temperature cure cycle (MRCC) for the 8552 Epoxy resin:

Manufacturer: Hexcel

Resin: 8552 Epoxy

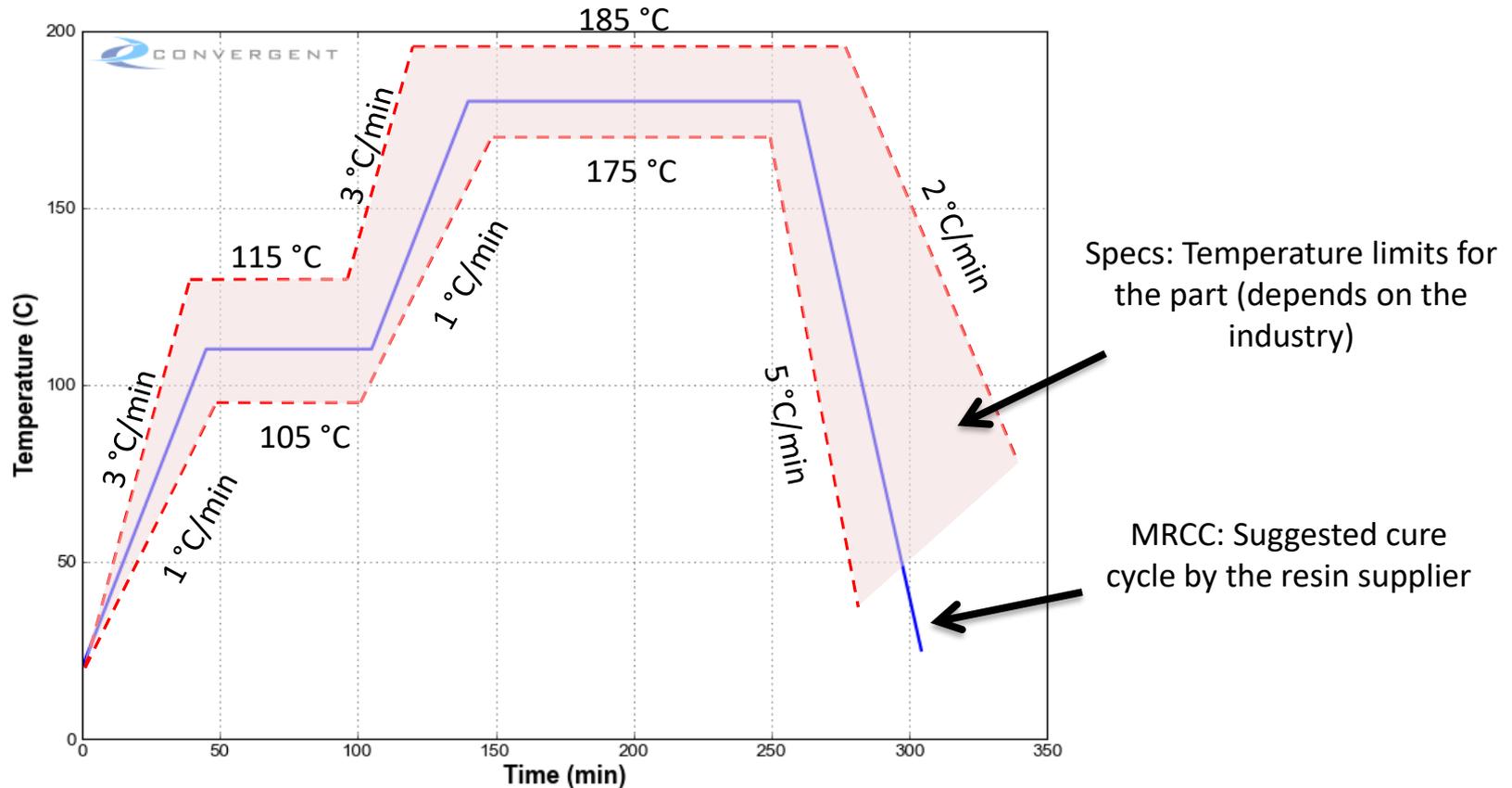
Usual Application: Aerospace

Autoclave Manufacturing



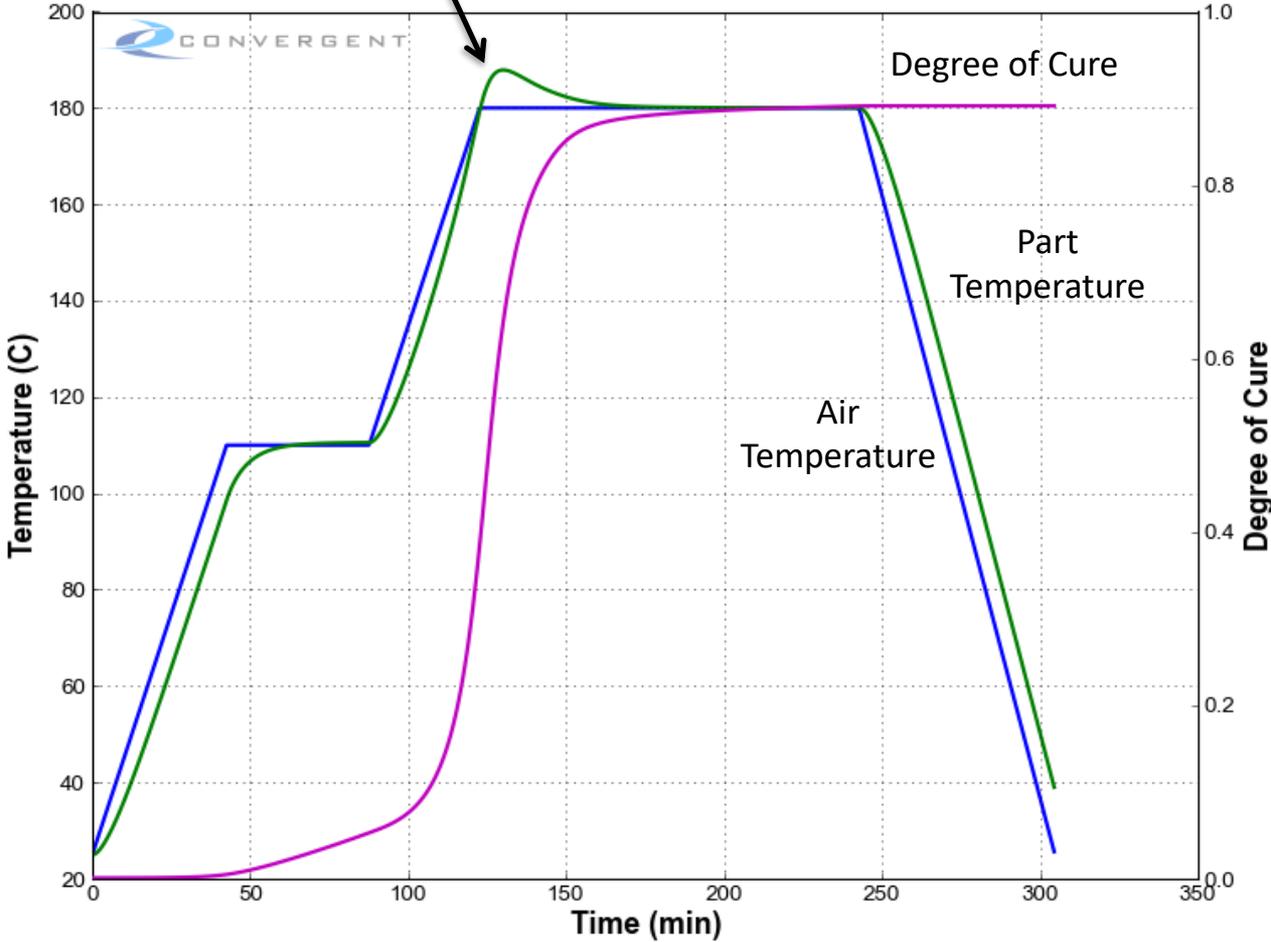
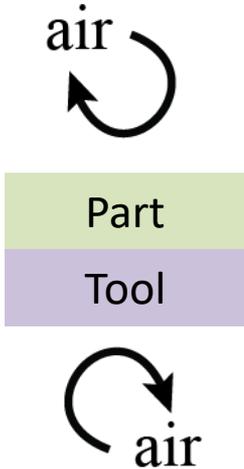
# Cure Cycles vs. Specs

- Temperature limits for the part are defined by specs (industry dependent). For a part to be approved, during the manufacturing process, temperature of every point in the part should be within the defined bound.



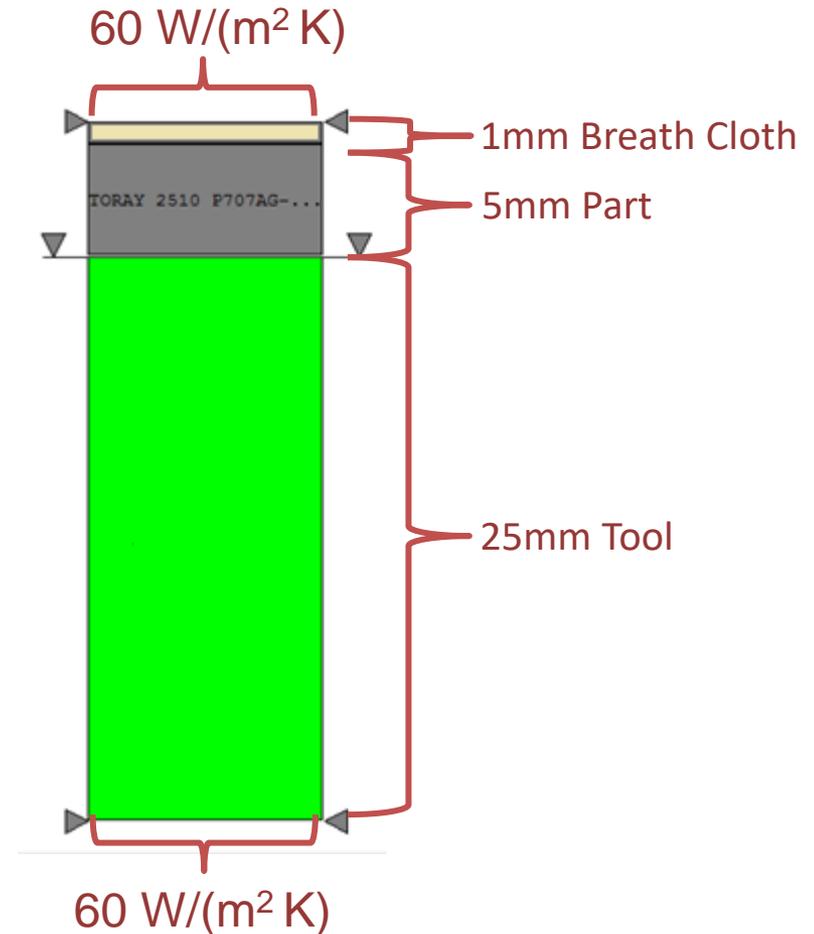
# Example: Acceptable Cure Cycle

Acceptable Exotherm < 185 °C



# Example Problem: Tooling Material Selection

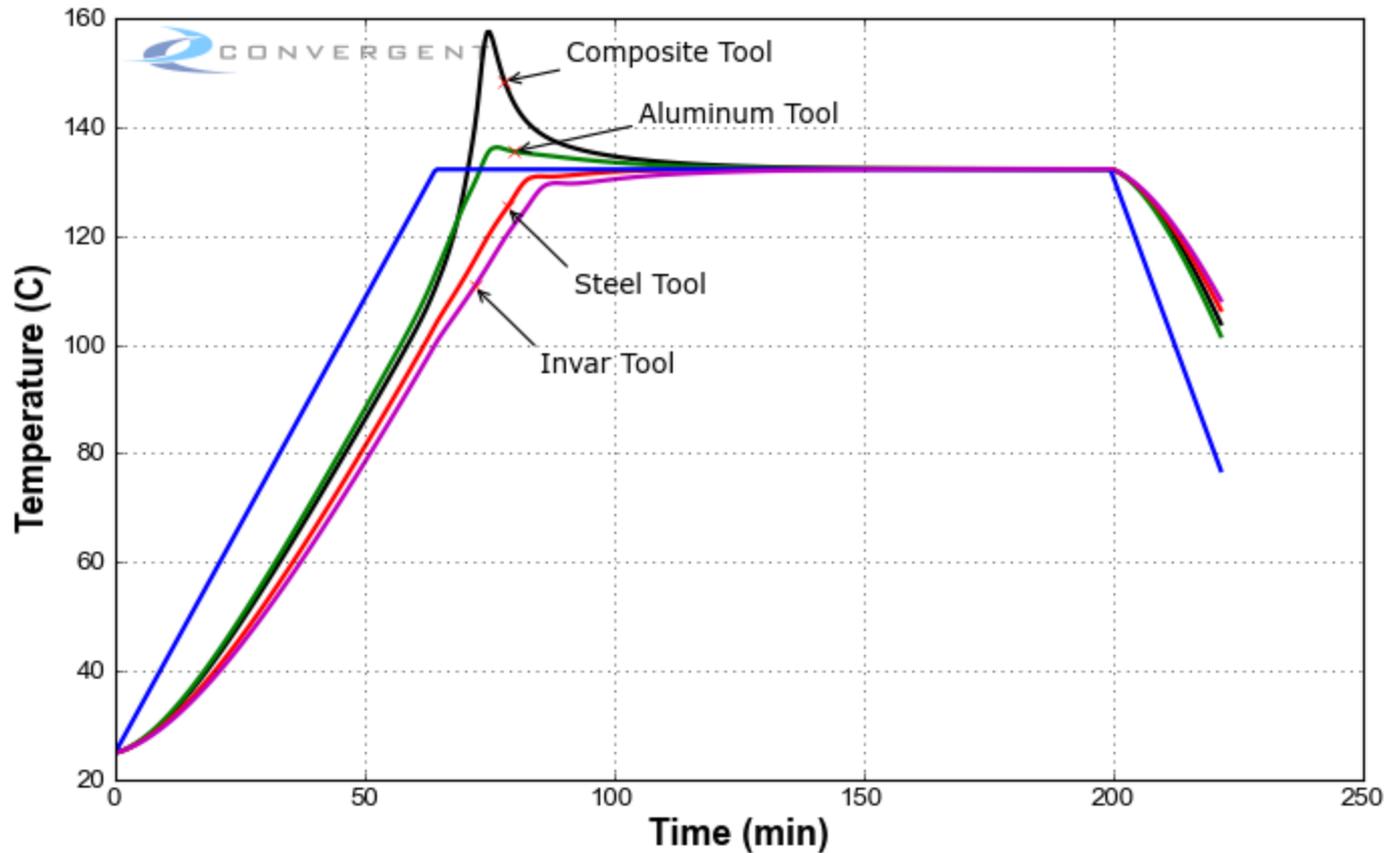
- Investigate the effect of the tooling material used to process a composite part made of Toray 2510 P707AG-15
- Select the tooling material that minimizes the exotherm and the thermal lag of the part



# Example Problem: Tooling Material Selection

---

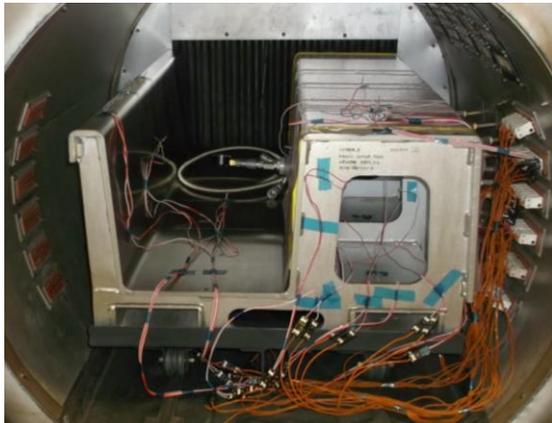
Part temperature (Top) summary:



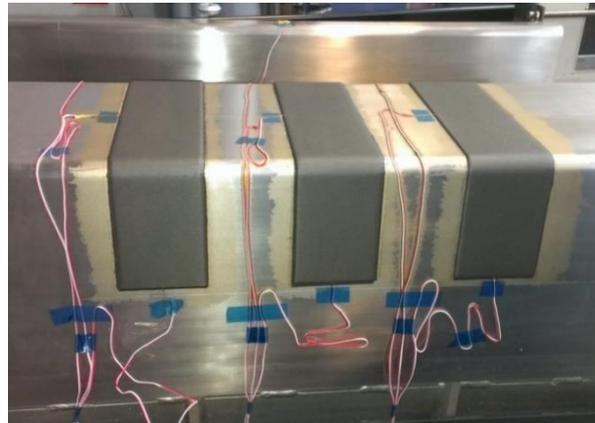
# Case Study: Fabricating C-Shape Parts in an Autoclave

- Thermocouple data provided by The Canadian Composite Manufacturing R&D Inc.
- C-shaped laminates @ 16, 32, and 64 ply thickness
- T800H/3900-2 and IM7/5320-1 plain weave CFRP prepreg
- Single hold cycle: 1.7°C/min to 2-hour hold (180 or 120°C)

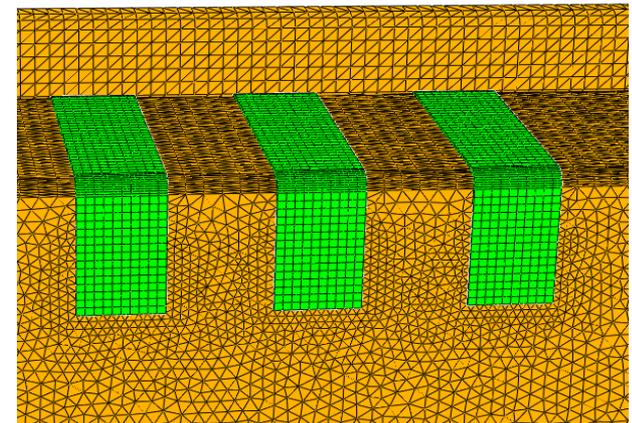
Rear of Vessel



Front (Door facing)

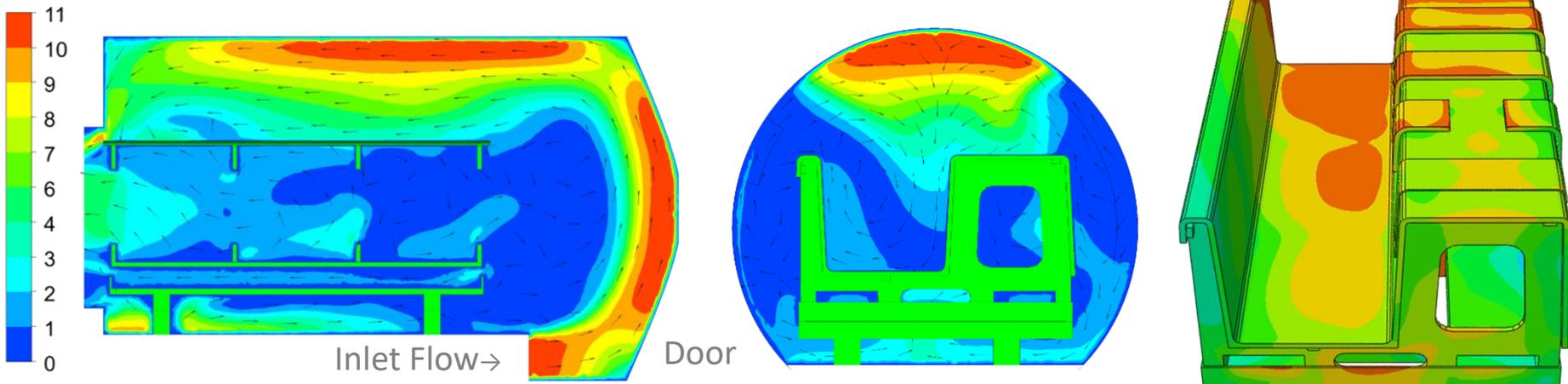
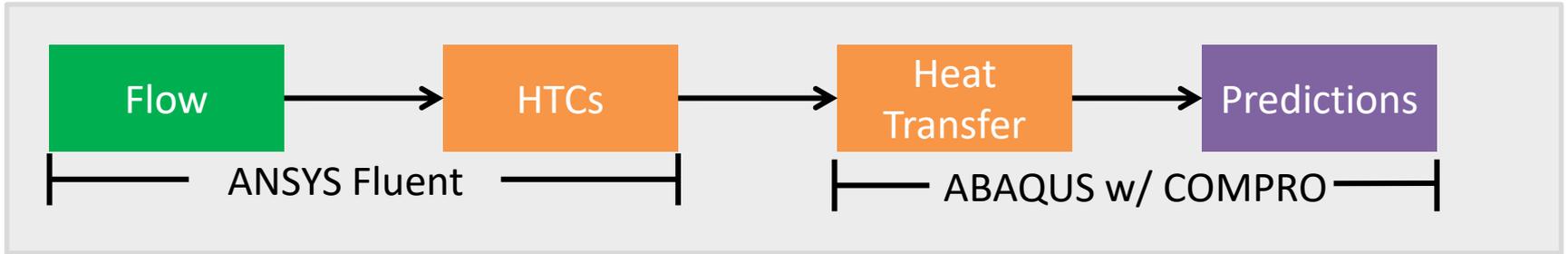


As Cured



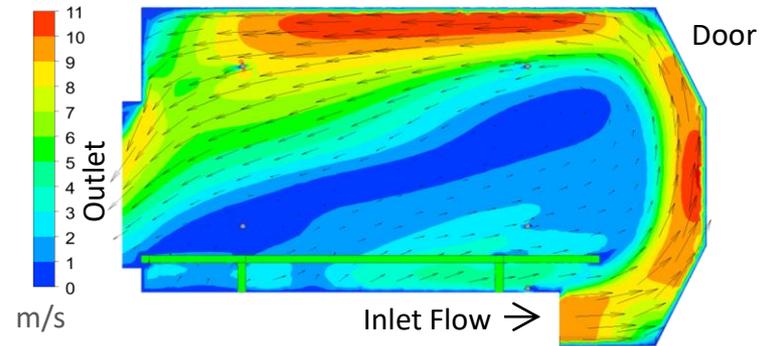
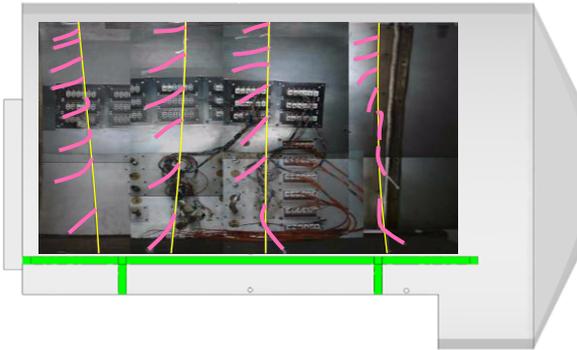
Digital Representation

# Simulation Approach

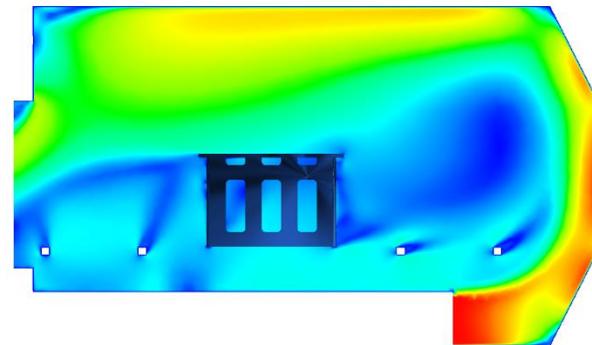


# Flow Simulation and Characterization

Empty Autoclave  
6.5 ft. (L) x 5 ft. (D)

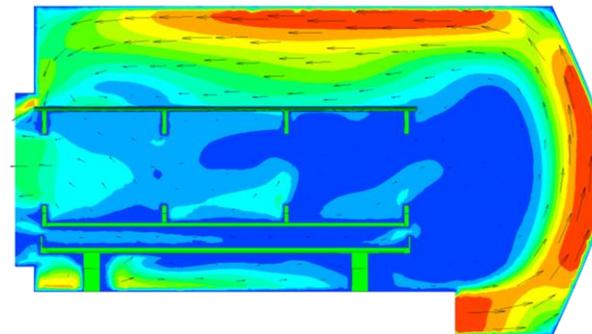


Small Tool  
1 ft. (L)



Relatively  
uniform HTC  
around tool

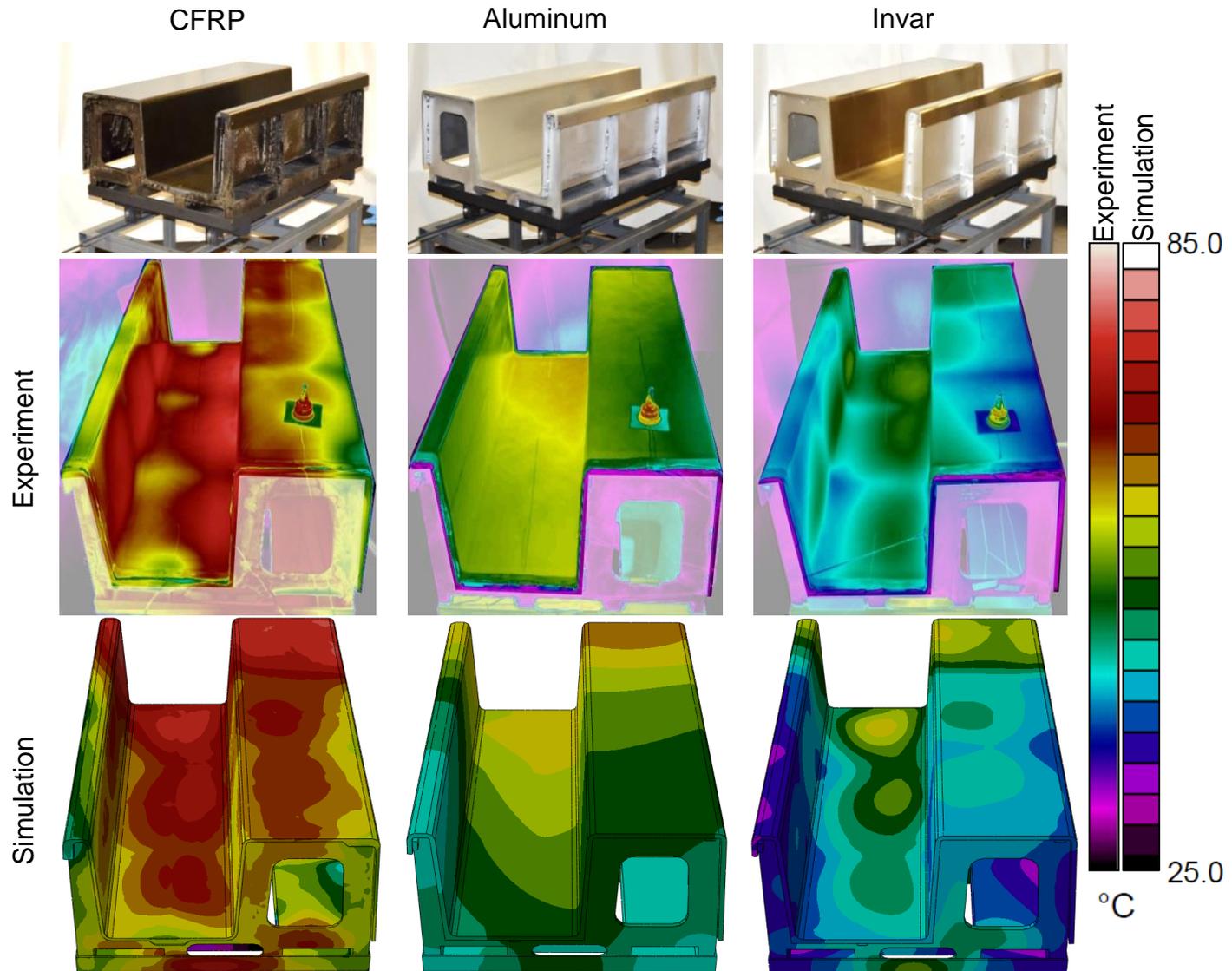
Large Tool  
4 ft. (L) x 2.3 ft.  
(W) x 1.6 ft. (H)



Non-uniform  
HTC around  
tool

CFD, RANS-based K-epsilon turbulence  
modelling, ANSYS Fluent

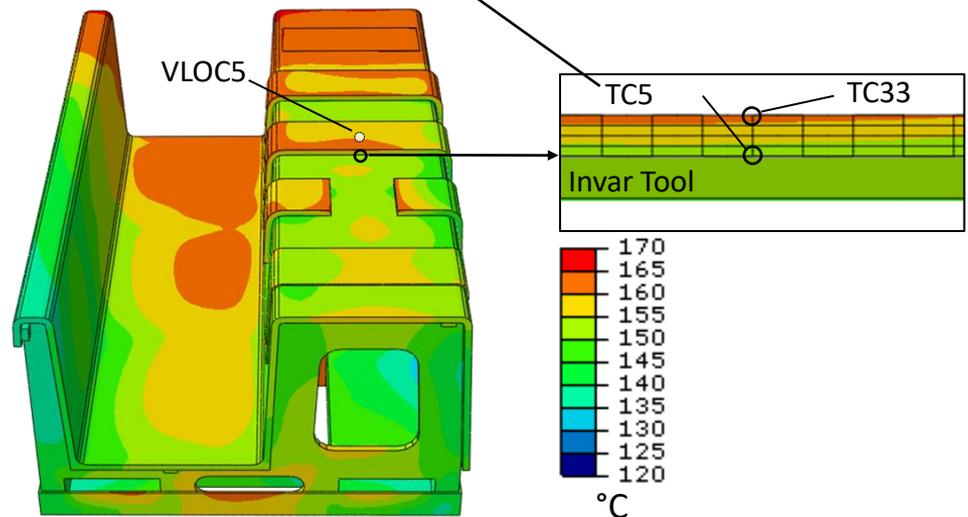
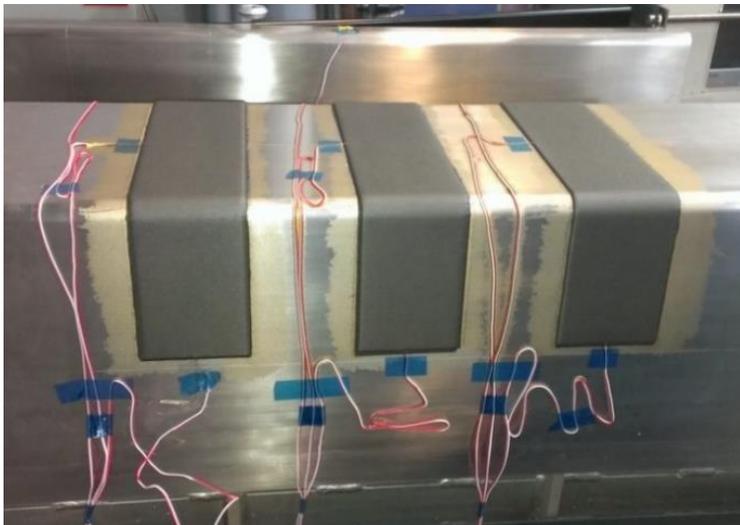
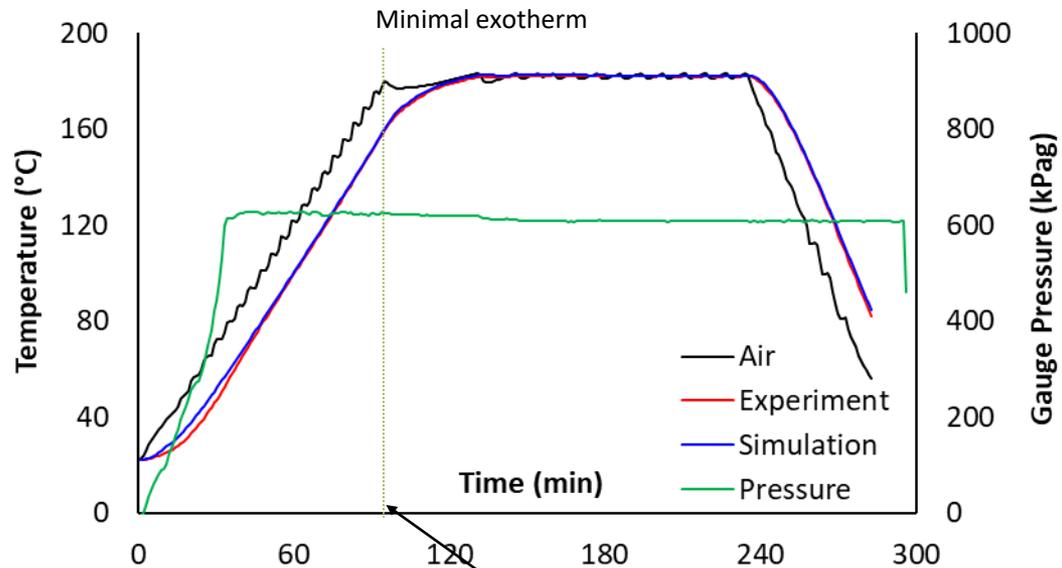
# Tool Response: IR Thermography vs. FE Simulation



5 °C/min to 100°C, Vacuum pressure only

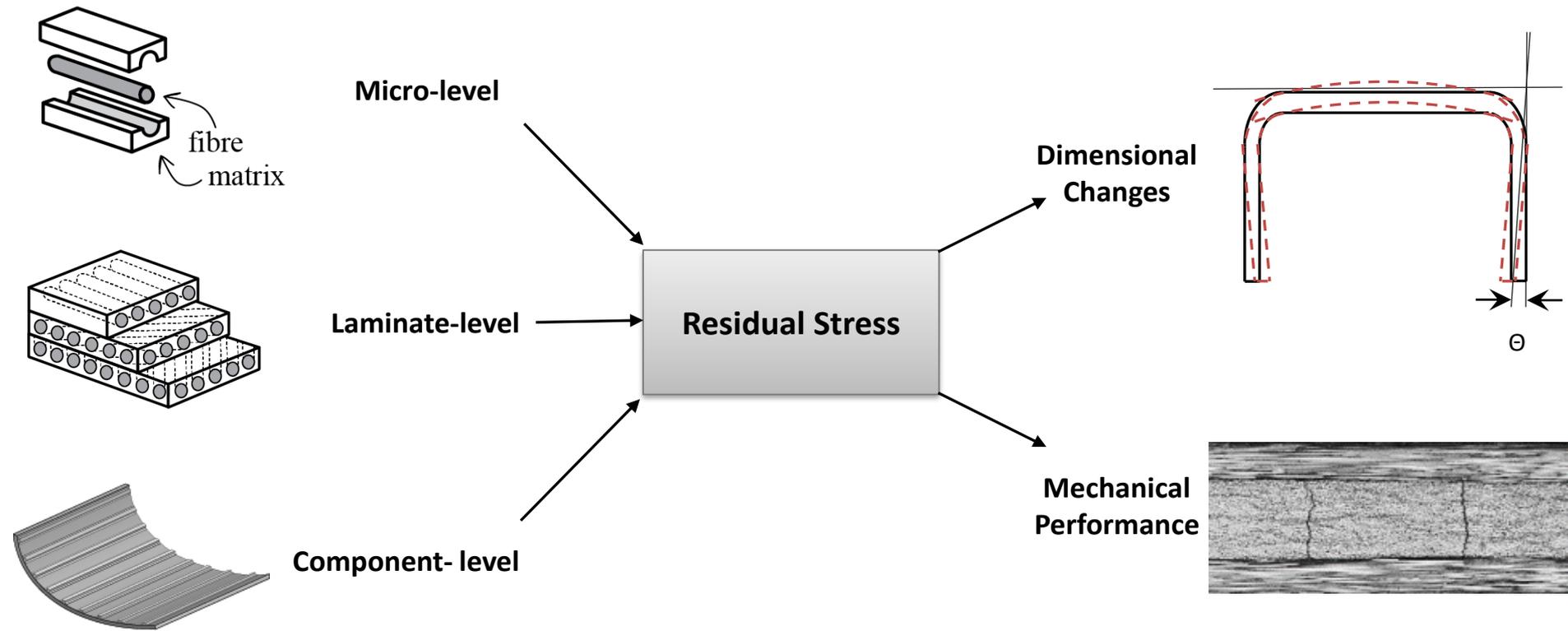
# Predicting Part Temperature History

- Thermocouple data by The Canadian Composite Manufacturing R&D Inc.
- Invar Tool
- C-shaped laminates @ 16, 32, and 64 ply T800H/3900-2 plain weave CFRP prepreg
- ABAQUS and COMPRO (material model)
- 1.7°C/min to 2 hour hold at 180 °C
- TC5 located at Tool – part interface



Front (Door facing)

# Stress and Deformation in Composites



Free strains

$$\epsilon_{free} = \epsilon_{thermal} + \epsilon_{phase-change} + \epsilon_{moisture}$$

# Constitutive Models

Thermo-Elastic (TE)

$$\sigma = E(T) \times (\varepsilon - \varepsilon_{free})$$

Pseudo-Viscoelastic (PVE)

Cure Hardening Instantaneous Linear Elastic (CHILE)

$$\sigma(t) = \int_0^t E'(T, \alpha) \frac{d\varepsilon(\tau) - d\varepsilon_{free}(\tau)}{d\tau} d\tau$$

Linear Viscoelastic (VE)

Linear viscoelastic, thermo-rheologically simple

$$\sigma(t) = \int_0^t E(E, \alpha, t - \tau) \frac{d\varepsilon(\tau) - d\varepsilon_{free}(\tau)}{d\tau} d\tau$$

Thermo-Viscoelastic (TVE)

Linear viscoelastic, thermo-rheologically complex

$$\begin{aligned} \sigma_x = E_e \varepsilon_x + a_F \int_0^\xi \Delta E(\xi - \xi') \frac{d\varepsilon_x}{d\xi'} d\xi' - \beta_e \Delta T \\ - a_F \int_0^\xi \Delta \beta(\xi - \xi') \frac{d\left(\frac{\Delta T}{a_F}\right)}{d\xi'} d\xi' \end{aligned}$$

Nonlinear Viscoelastic (NVE)

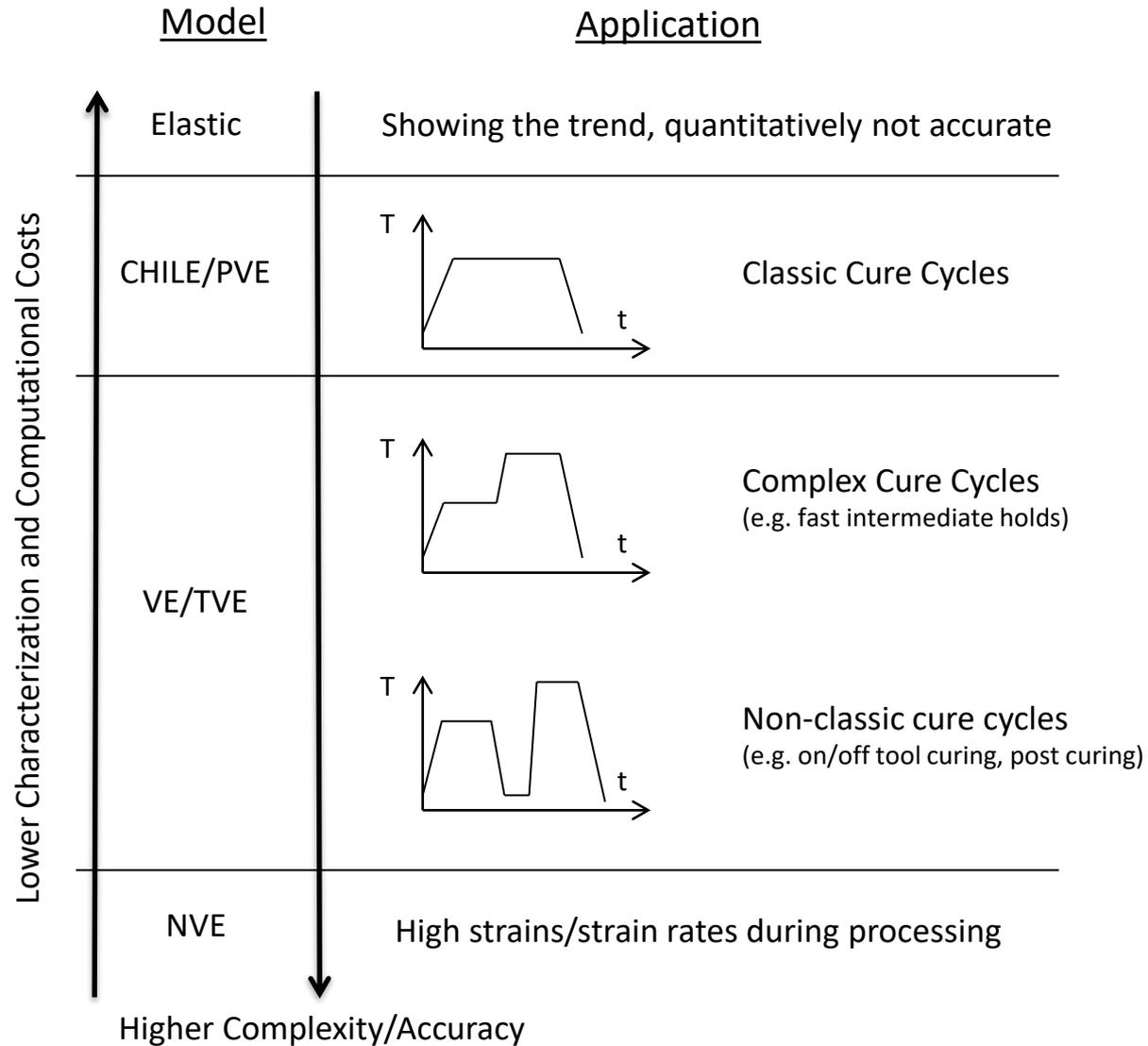
Nonlinear viscoelastic, thermo-rheologically complex

Lower  
Computational  
Cost

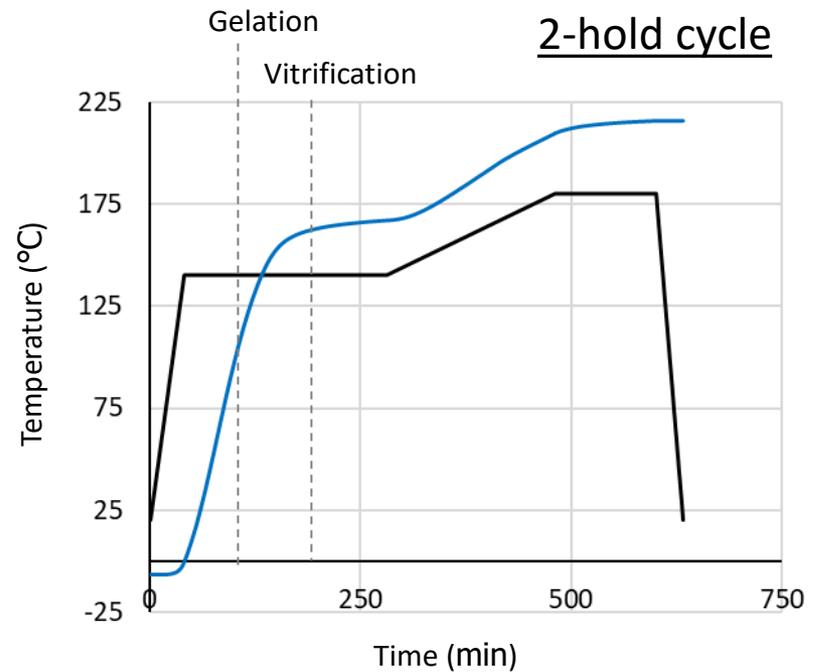
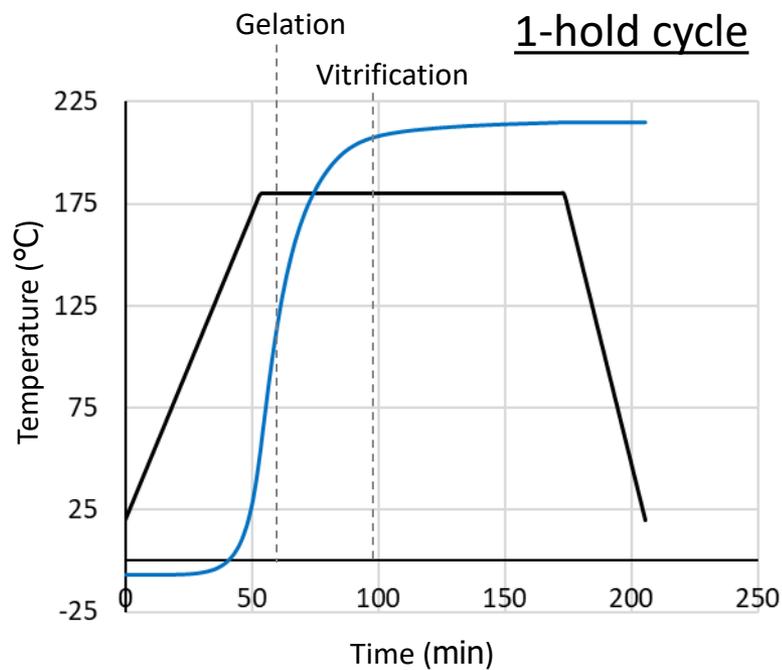
Lower  
Characterization  
Cost

Higher  
Fidelity

# Constitutive Models



# Effect of Cure Cycle: AS4/8552

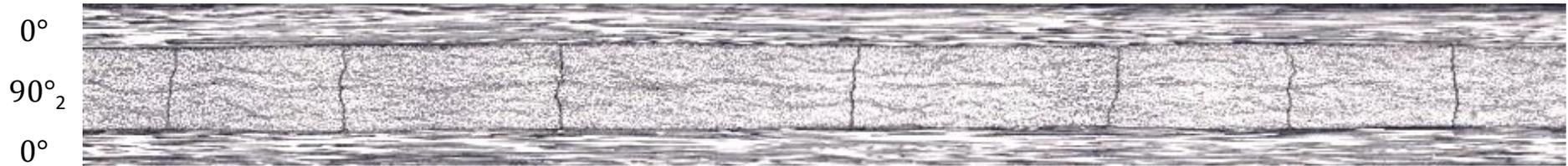


— Temperature

— Glass Transition Temperature ( $T_g$ )

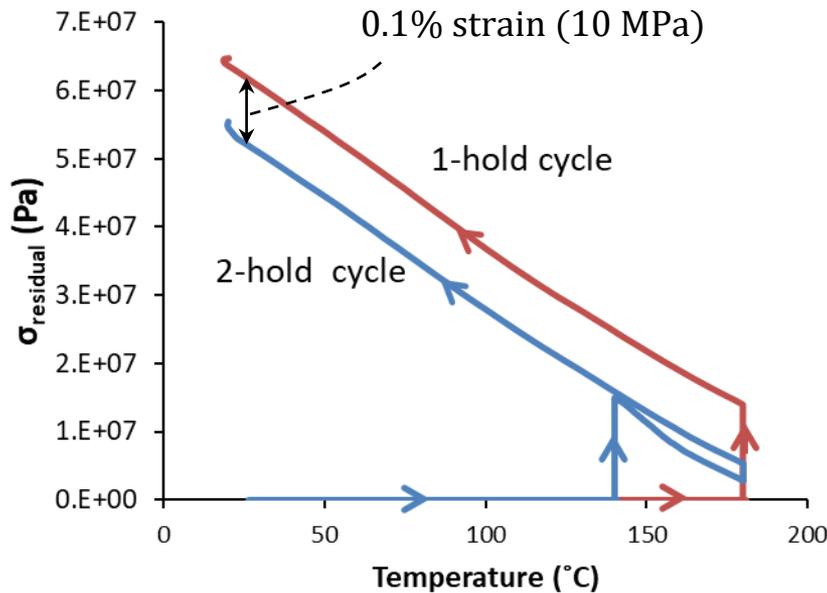
Cycle	heat-up	dwll	heat -up	dwll	cool-down
1	3 °C/min			2 hrs. @ 180 °C	5 °C/min
2	3 °C/min	4 hrs. @ 140 °C	0.2 °C/min	2 hrs. @ 180 °C	5 °C/min

# Residual Stress in Cross-Ply Laminates

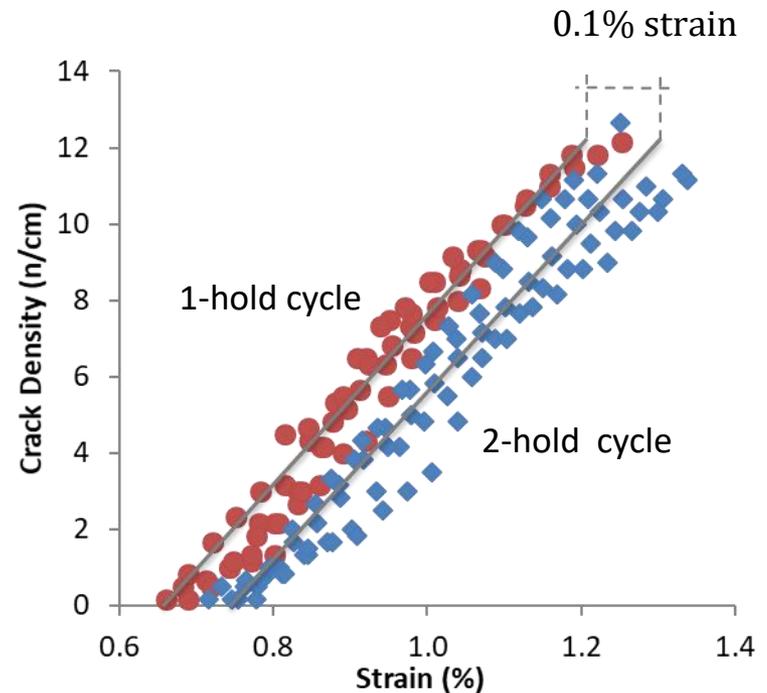


Predictions of Residual Stresses in  $[0/90]_s$  Laminates

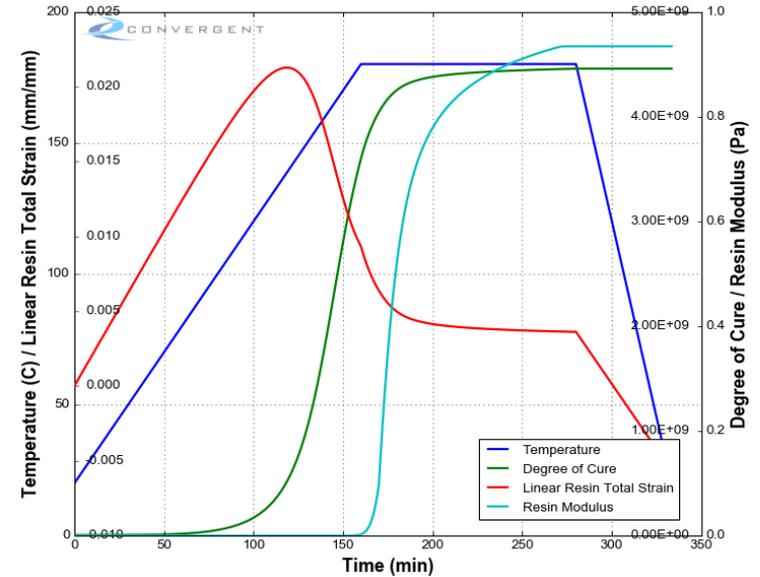
$$\sigma_{residual} = \frac{-E_0 t_0}{\sum E'_i t_i} \int E'_{90} d\varepsilon_{free}$$



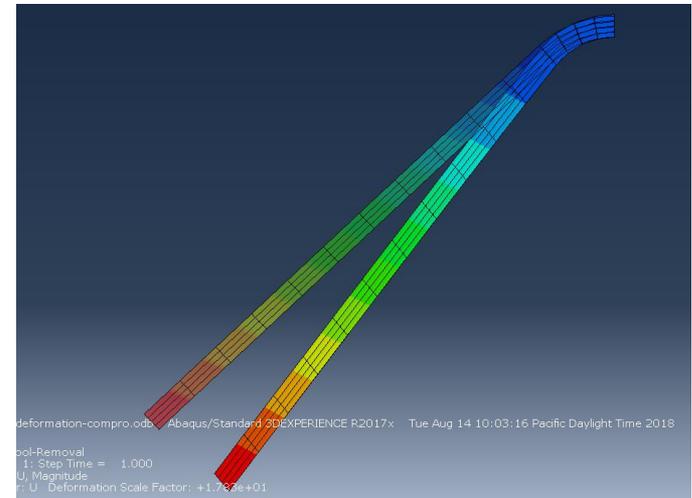
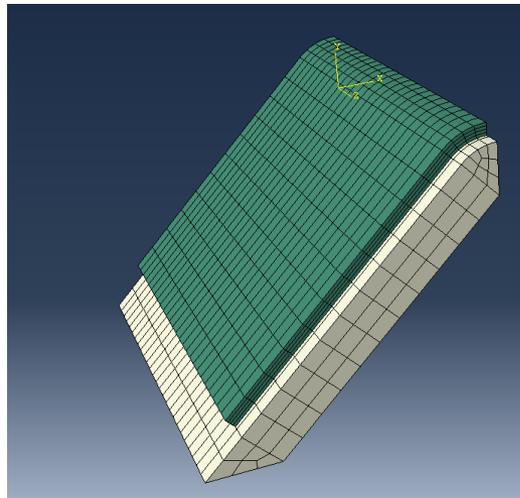
Matrix Cracking Tests on  $[0/90]_s$  Laminates



# Part Deformation Simulation

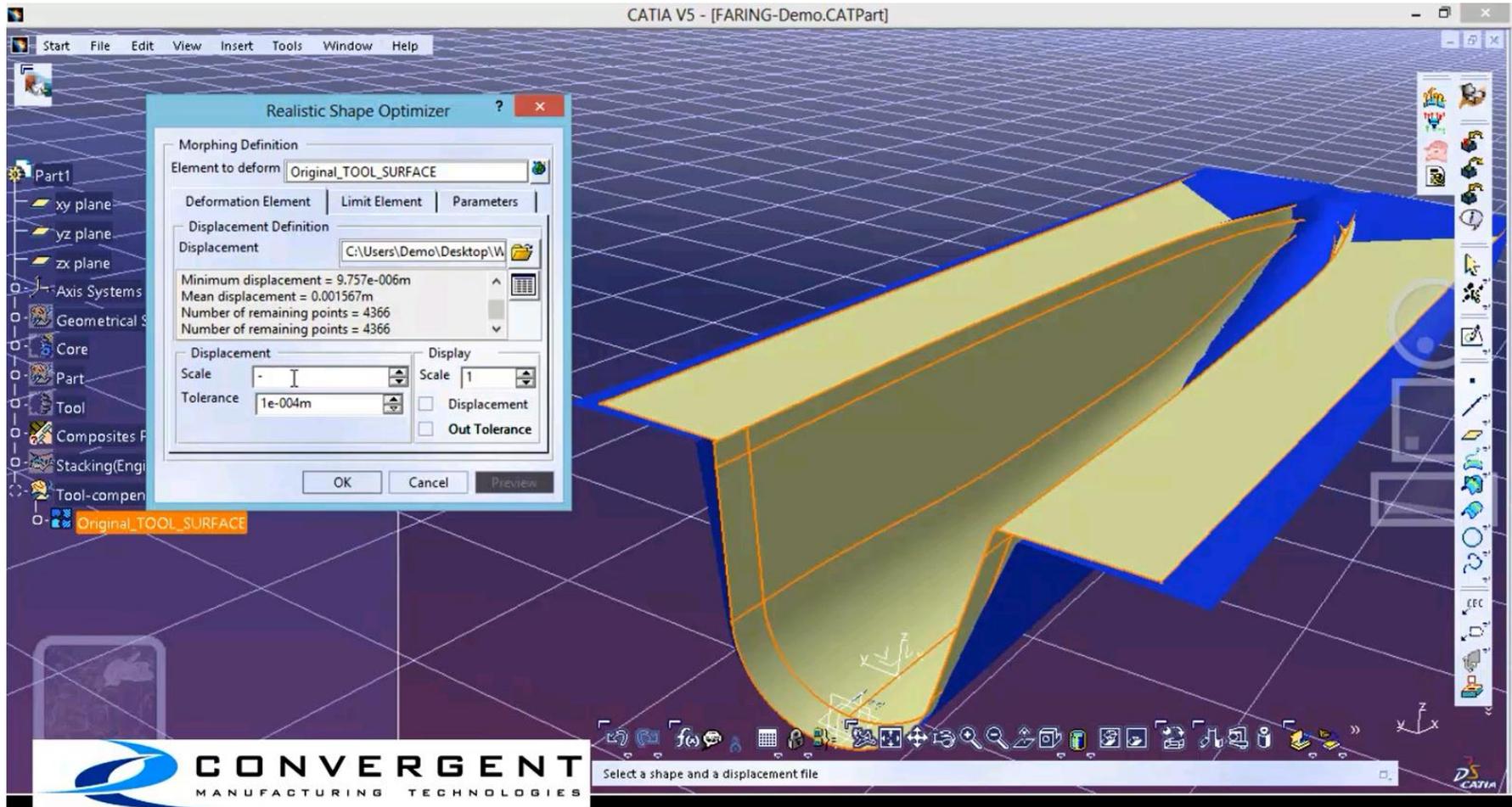


ABAQUS and COMPRO  
NCAMP material model  
Element type C3D20  
frictionless tool-part



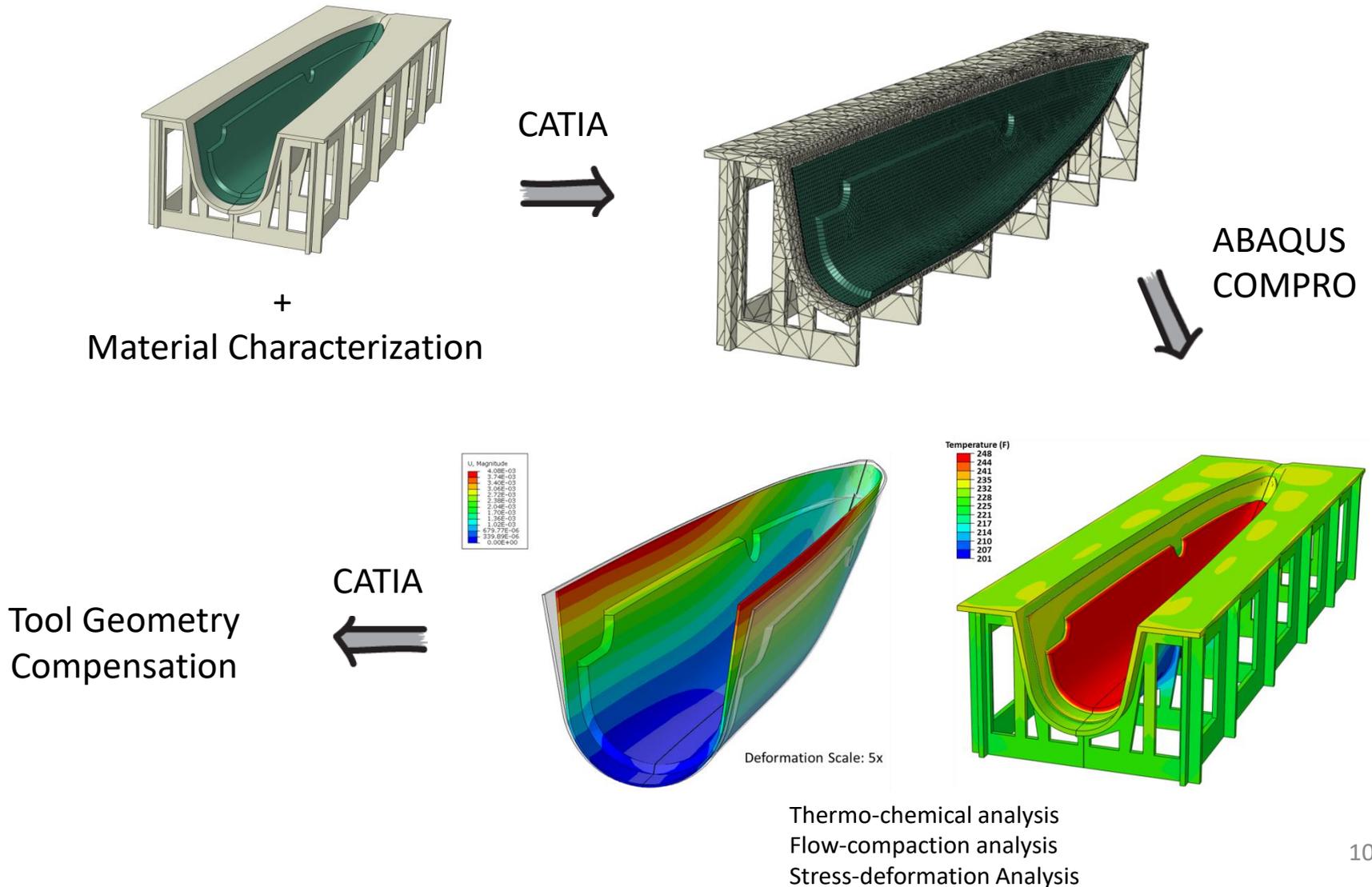
# Process-Induced Deformation by COMPRO

– Tool geometry compensation in CATIA



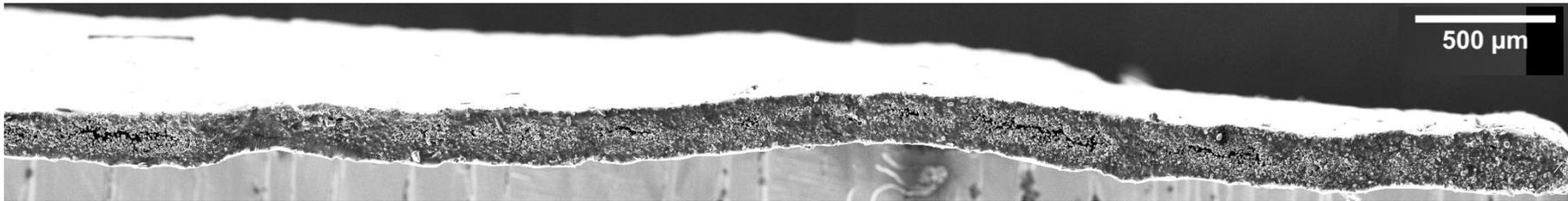
# Process-Induced Deformation by COMPRO

– Current tool geometry compensation workflow

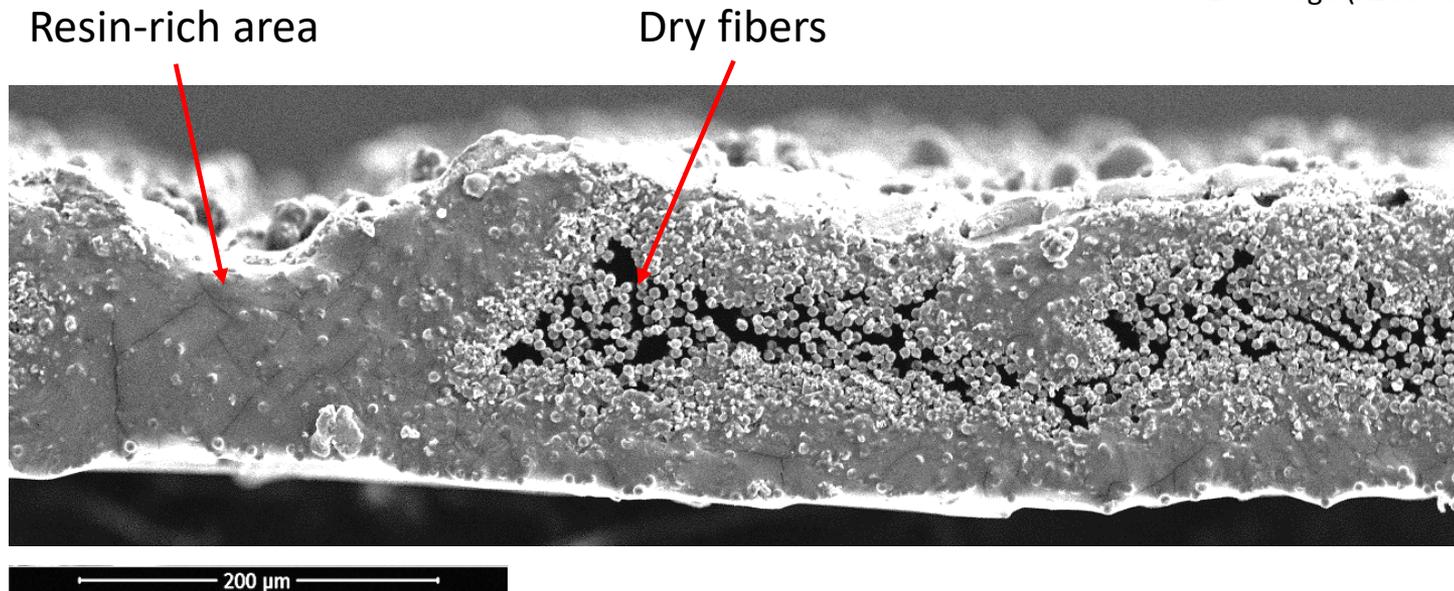


# Initial Prepreg Condition

- HEXCEL AS4/8552 carbon-epoxy UD prepreg
- Made using the hot melt process



AS4/8552 cut at room temperature  
SEM Image (X200 Magnification)



# Effect of Pressure

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Uncured Prepreg  
(Room Conditions)

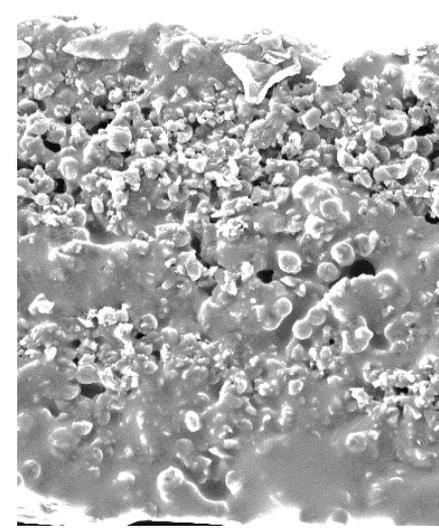
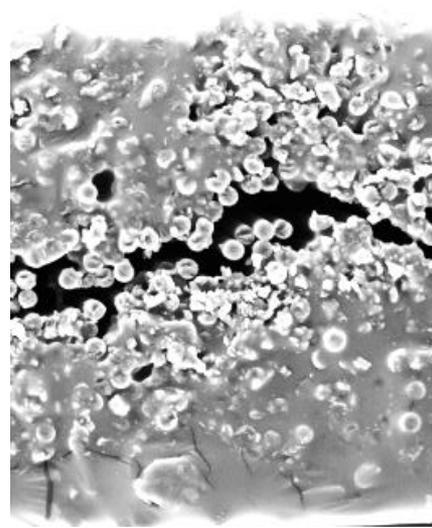
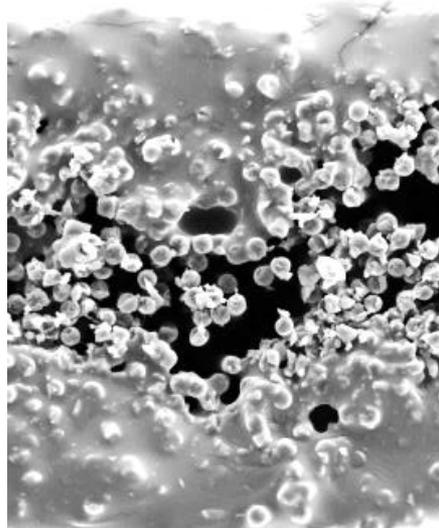
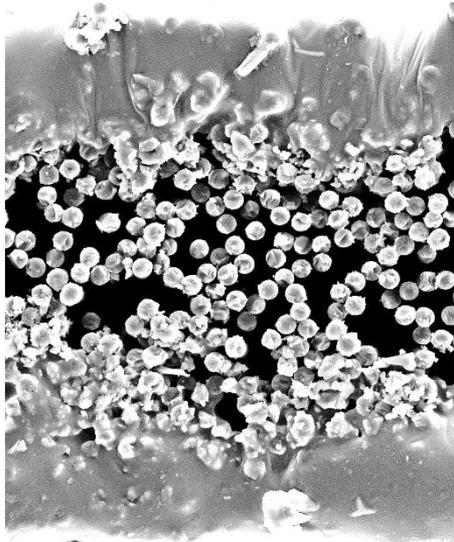
15 min @ 40 °C (104 °F)  
(Under vacuum)

15 min @ 60 °C (140 °F)  
(Under vacuum)

15 min at 80 °C (176 °F)  
(Under vacuum)

Thickness = 200 – 220  $\mu\text{m}$

Thickness = 165 – 185  $\mu\text{m}$



100  $\mu\text{m}$

# Consolidation Under Vacuum Pressure

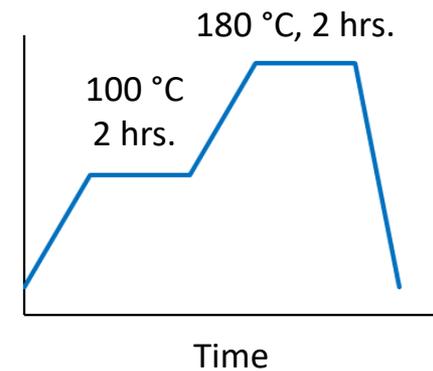
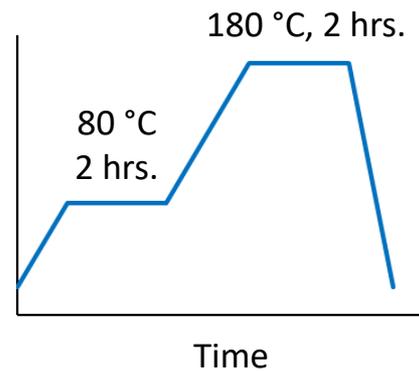
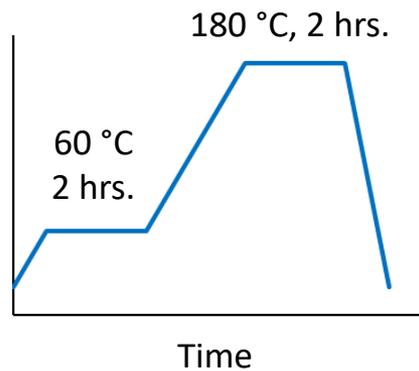
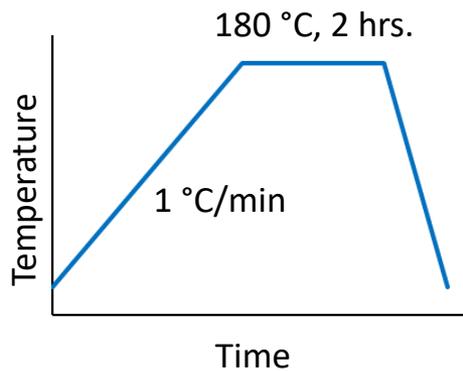
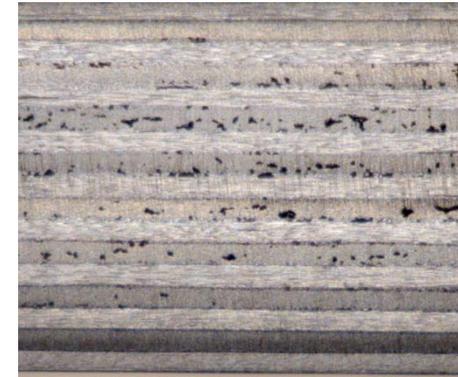
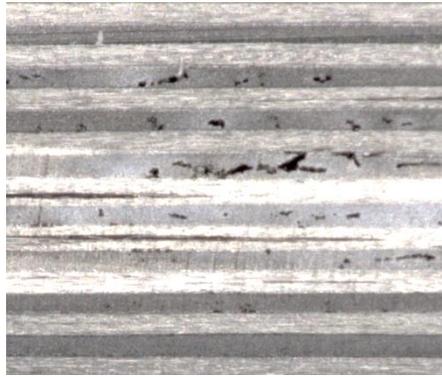
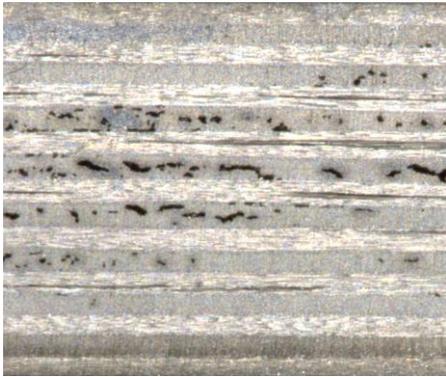
- Effect of Cure Cycle on Porosity:

Ply thickness = 0.196 mm  
( $\approx$  7% porosity)

Ply thickness = 0.189 mm  
( $\approx$  3.5% porosity)

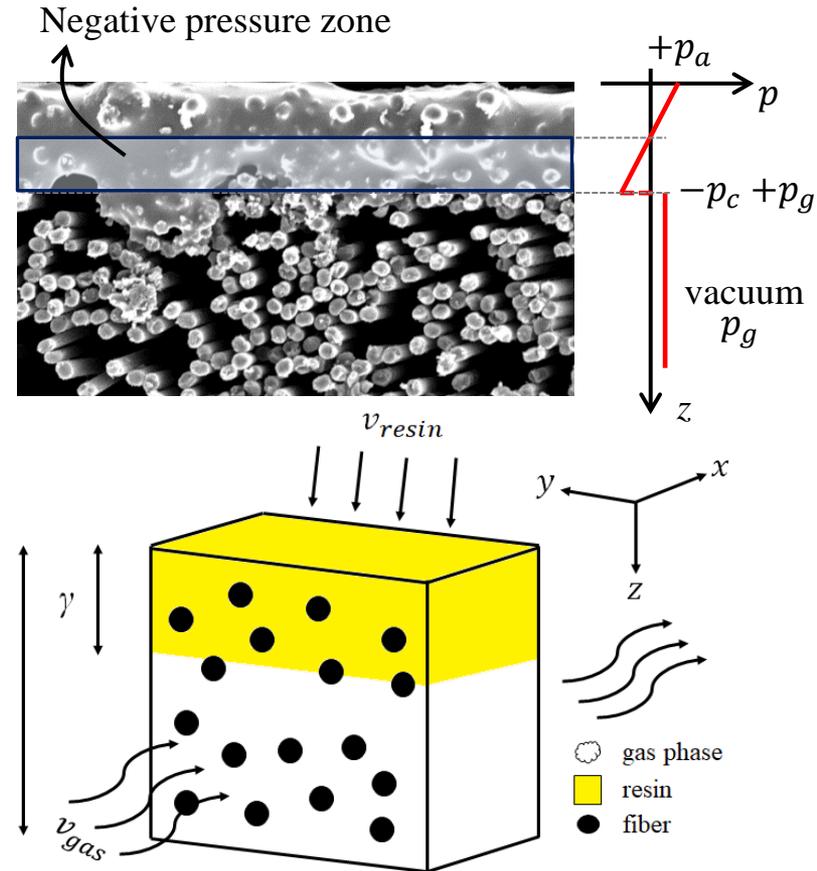
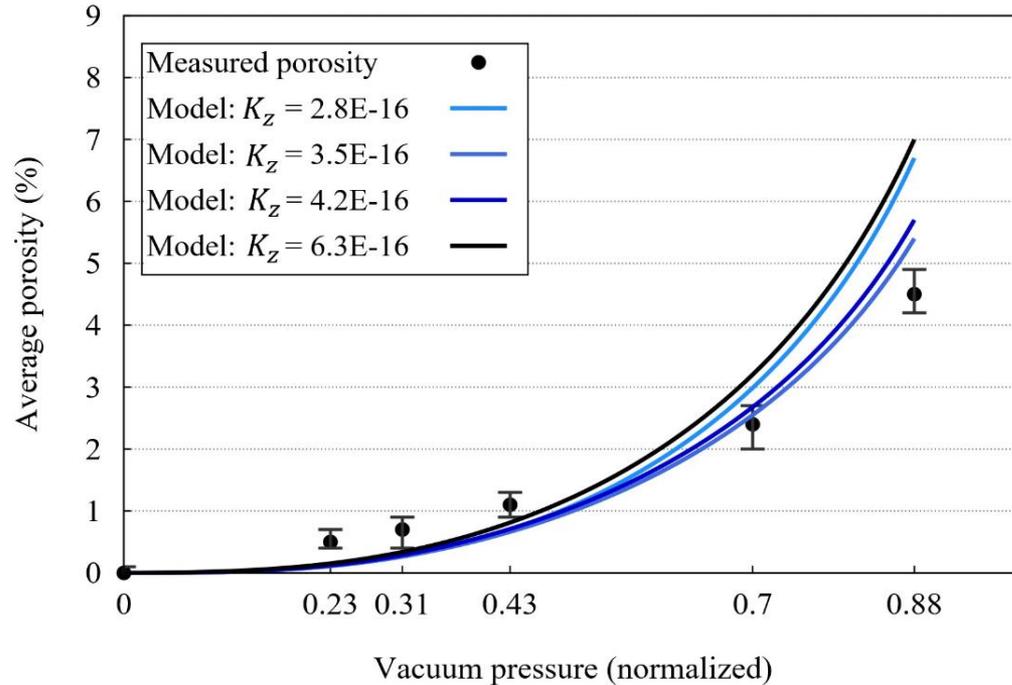
Ply thickness = 0.182 mm  
( $<$  0.5% porosity)

Ply thickness = 0.192 mm  
( $\approx$  5% porosity)



# Coupled Gas/Resin Transport Model

MTM45-1/5HS CF2426A by Cytec Solvay



$$\begin{cases} \varphi \frac{\partial P_g}{\partial t} = \frac{K_{x0}}{\mu_g} \frac{\partial}{\partial x} \left( \left( 1 + \frac{b}{P_g} \right) P_g \frac{\partial P_g}{\partial x} \right) + \frac{K_z}{h^2 \mu_r (1 - V_f)} \frac{P_g (P_a - P_g + P_c)}{(1 - \varphi)} \\ \frac{d\varphi}{dt} = - \frac{K_z}{h^2 \mu_r (1 - V_f)} \frac{P_a - P_g + P_c}{(1 - \varphi)} \end{cases}$$

Gas flow in x direction

Resin flow in z direction

# Evolution of properties during processing and characterization methods

2021-06-09

Navid Zobeiry, PhD (navidz@uw.edu)  
Assistant Professor  
Materials Science & Engineering  
University of Washington



***BE BOUNDLESS***

**MULE**

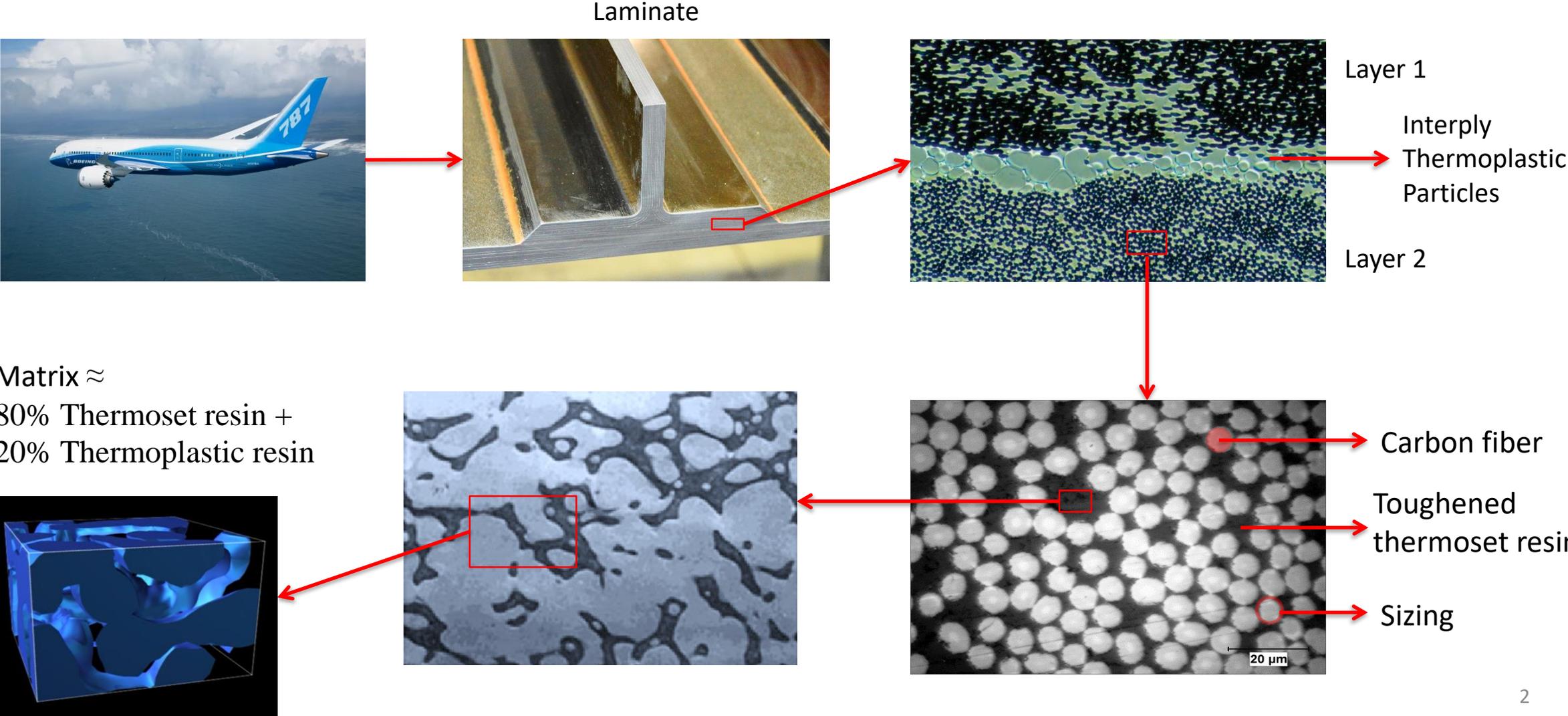
**AIDAA** EDUCATIONAL SERIES  
& ACADEMY

 Politecnico  
di Torino

**W**

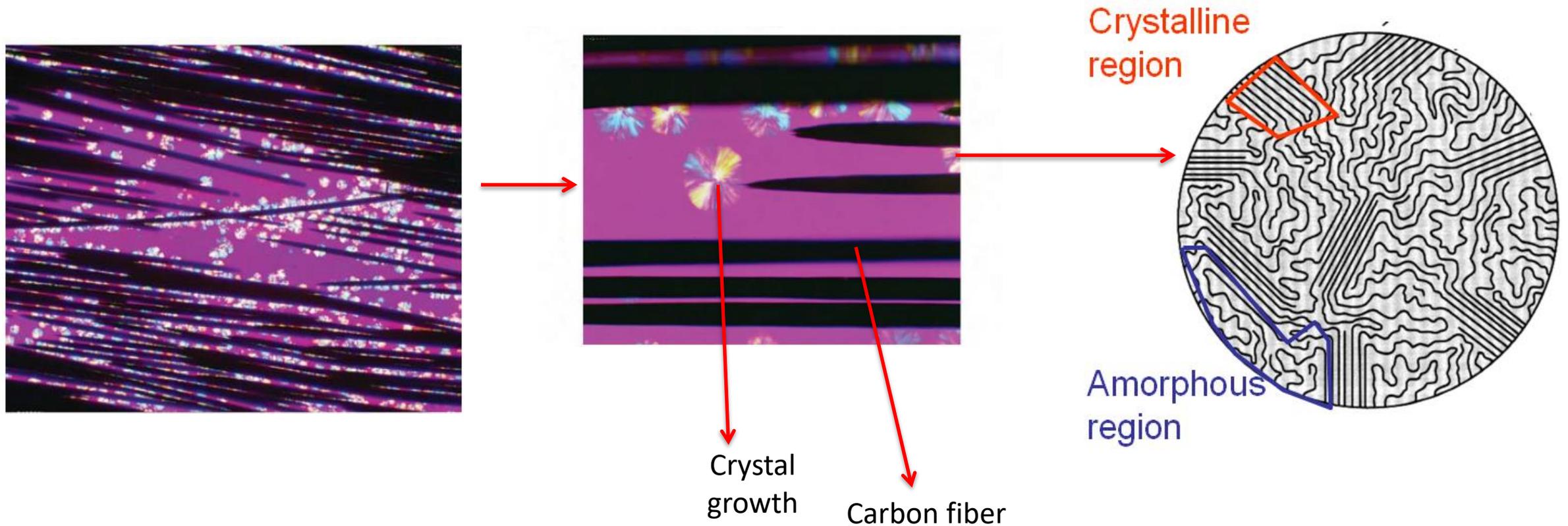
# Aerospace Composite Materials

## Typical toughened thermoset composites



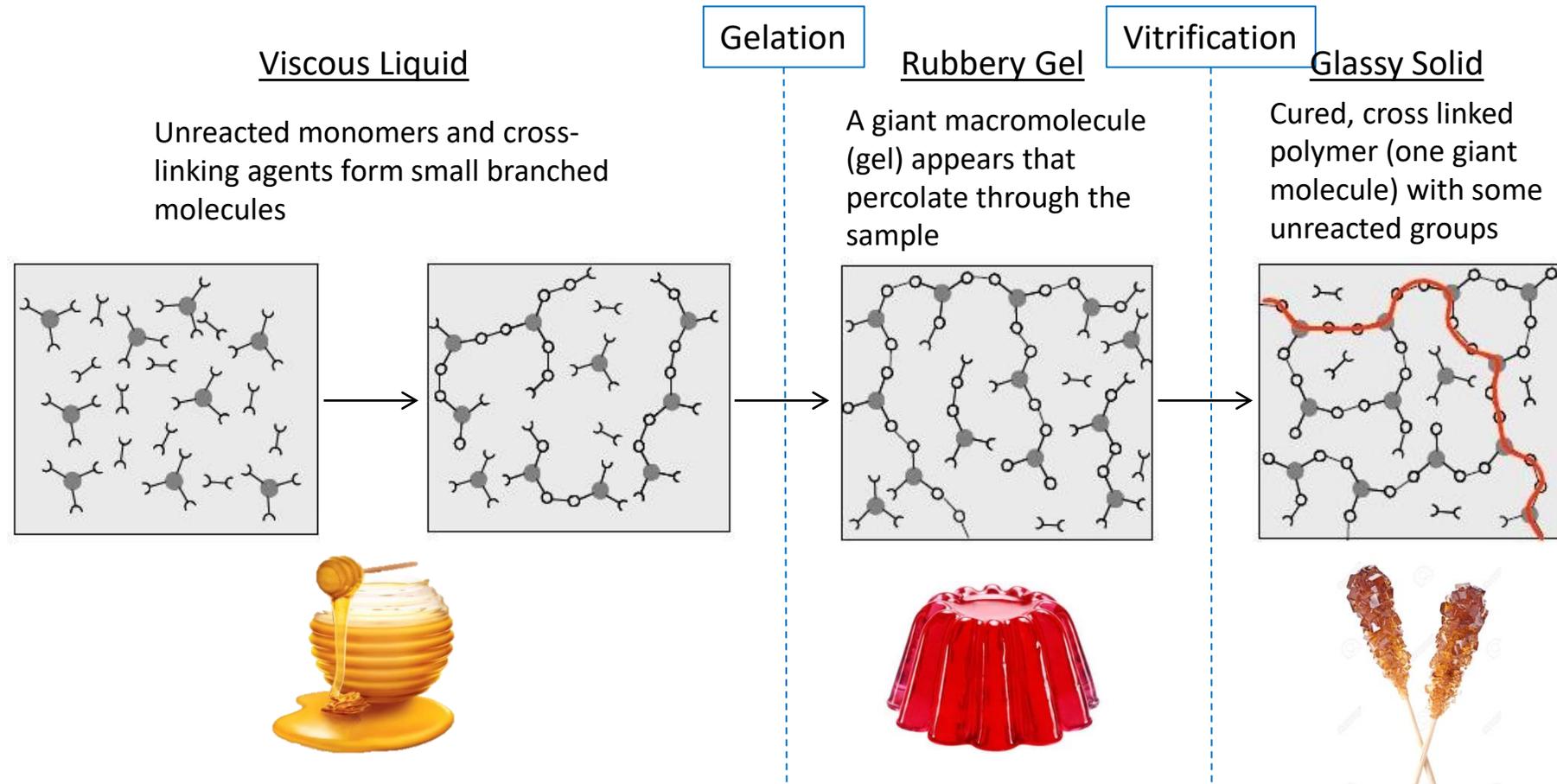
# Aerospace Composite Materials

Typical semi-crystalline thermoplastic composites



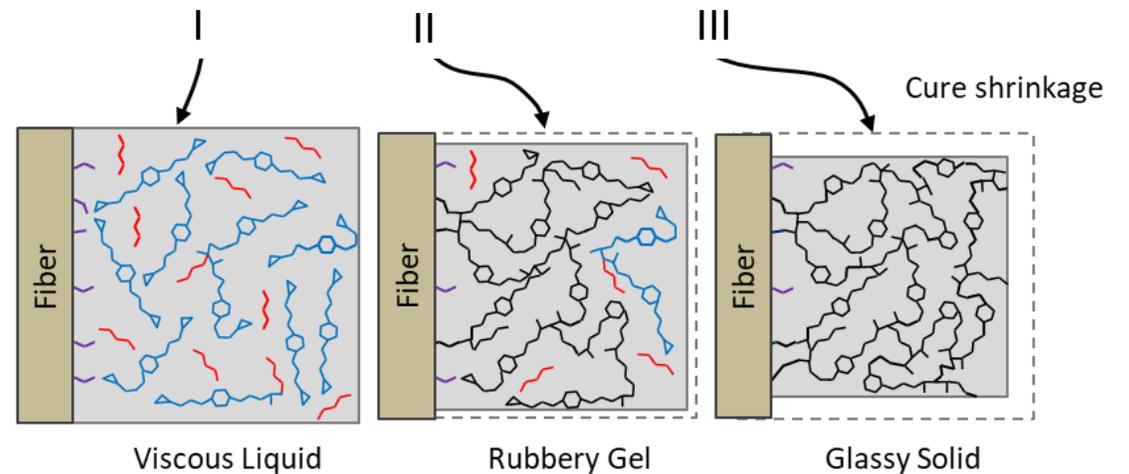
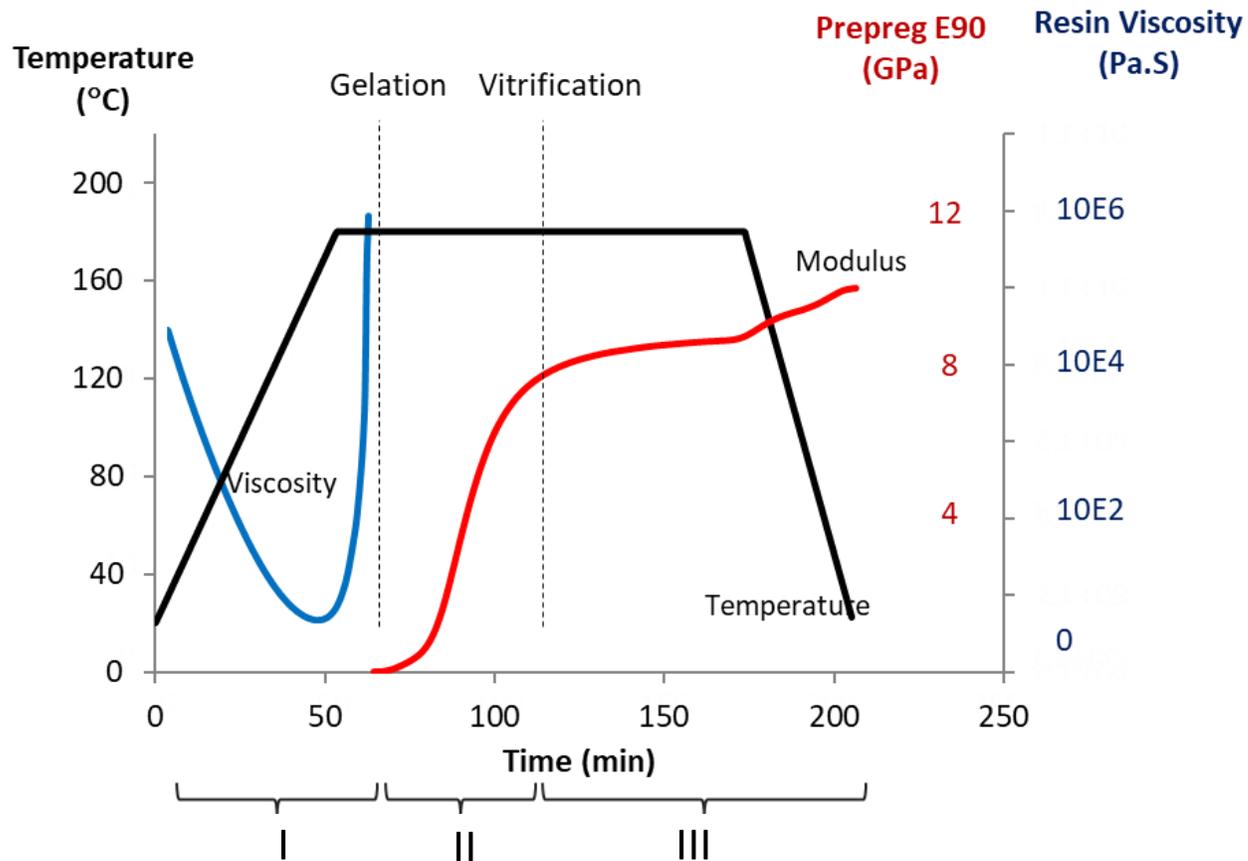
# Curing Process in Thermosets

- During polymerization, material goes through different phases.
- At gelation, it changes from a viscous liquid to a rubbery gel
- As the curing advances, the rubbery gel transforms into glassy solid at vitrification



# Curing Process in Thermosets

- Example: HEXCEL AS4/8552 (aerospace grade toughened epoxy prepreg)
  - 65% by weight AS4 carbon fiber, 35% by weight 8552 resin
  - 8552 resin:  $\approx 75\%$  epoxy +  $25\%$  PES



# Thermosets and Heat

- If heated after cured, thermosets soften initially and then transform into char at very high temperatures.

Heat



## Glassy Solid

Cured, cross linked polymers



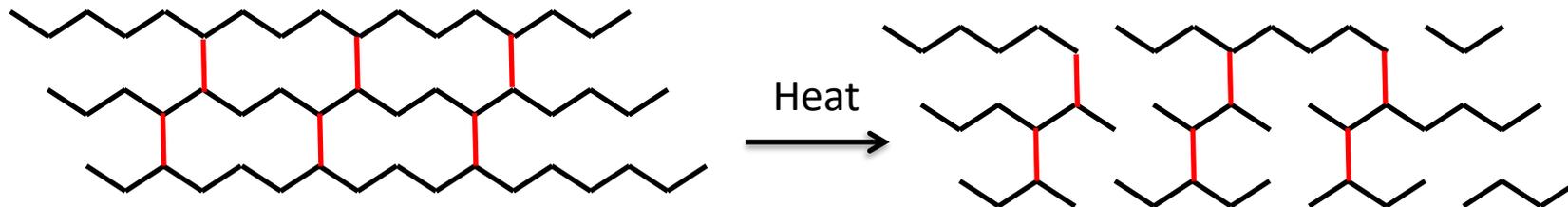
## Rubbery Gel

Increase of chain mobility when temperature goes above  $T_g$



## Burnt

Oxidation and breaking of covalent bonds

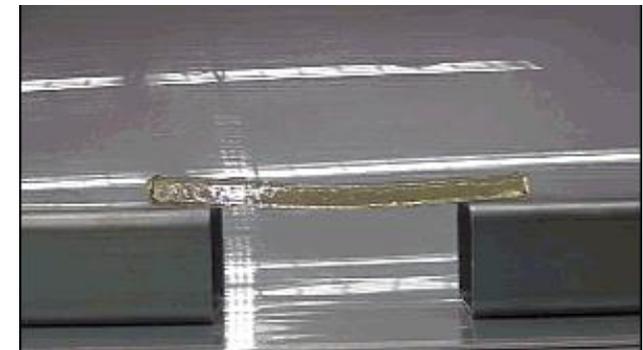


Degradation

## Glassy (thermoelastic)



## Rubbery (viscoelastic)

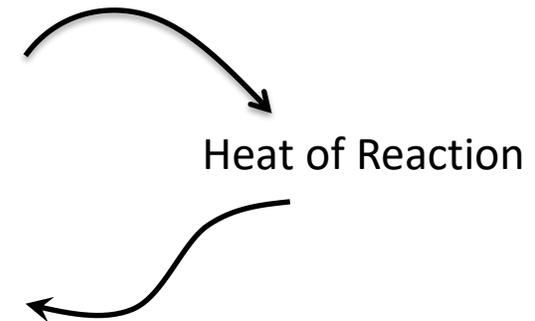
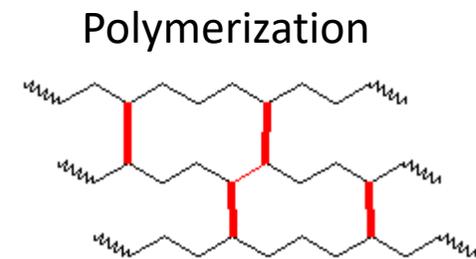


# Heat of Reaction

- Curing of thermoset resins is an exothermic and heat is generated. The heat of reaction during polymerization is measured using a Differential Scanning Calorimeter (DSC) equipment.
- DSC measures how much energy/heat comes out of the reaction for a small resin sample



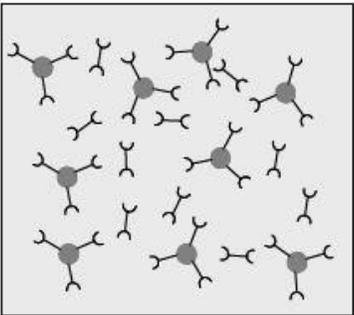
Loading a composite sample



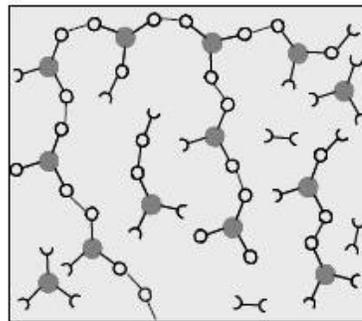
# Degree of Cure

- Using heat of reaction, Degree of cure (DOC) is calculated.
- DOC is defined between 0 and 1 and is an indication of how far the crosslinking is advanced in a thermoset resin.

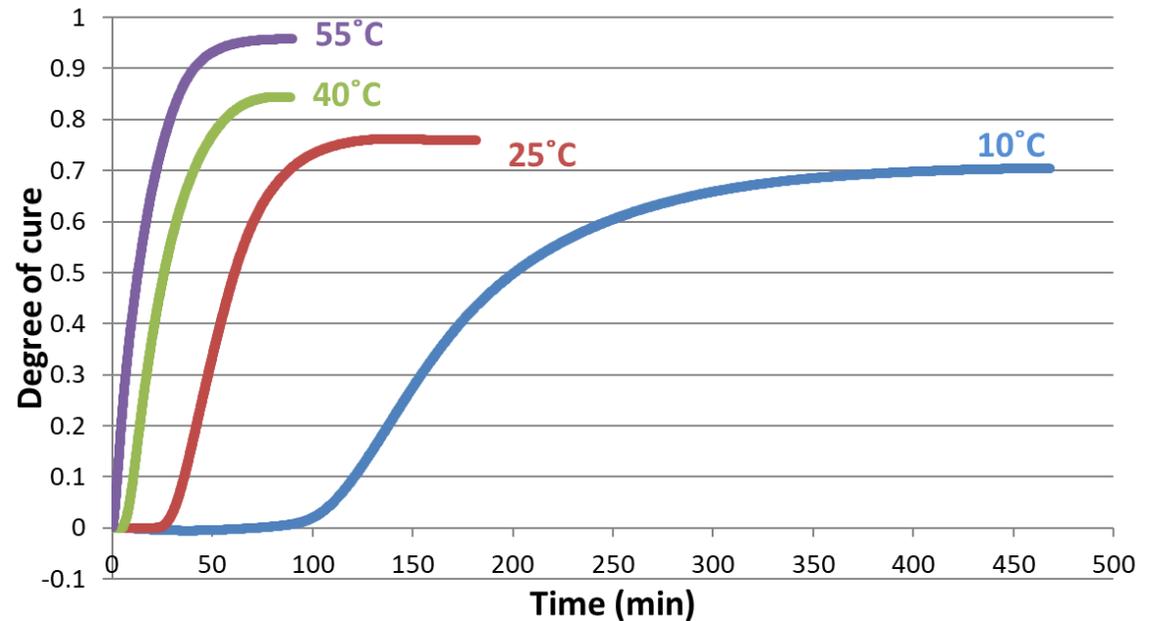
Low Degree of Cure  
Low mechanical properties



High Degree of Cure  
High mechanical properties

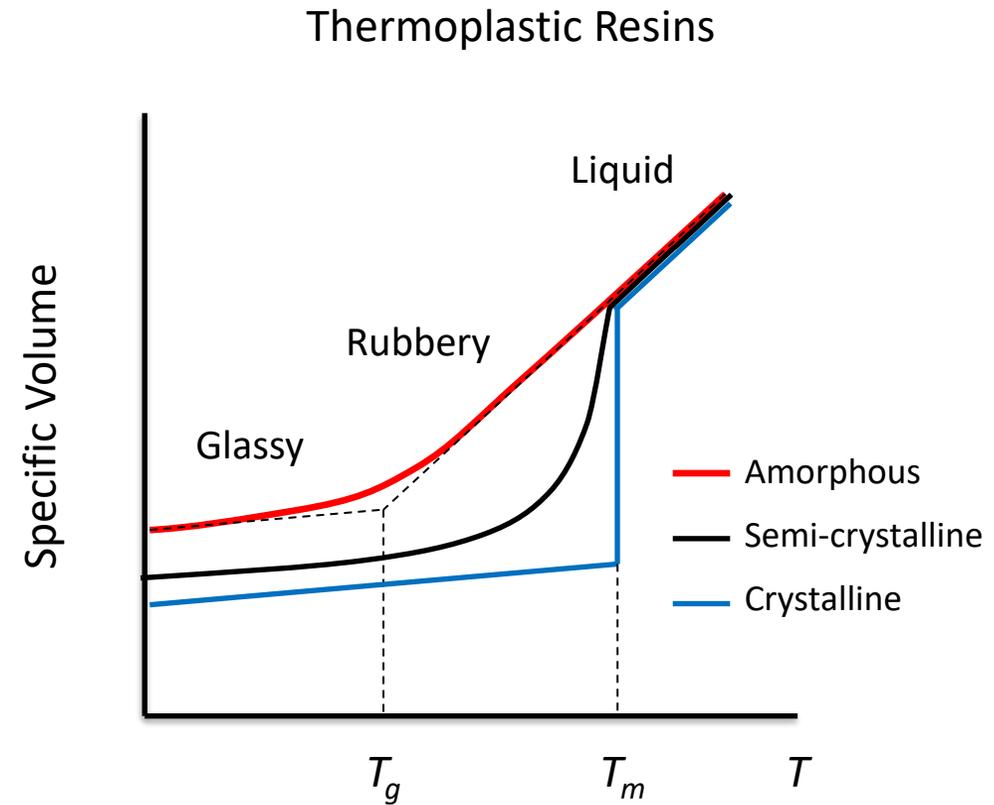
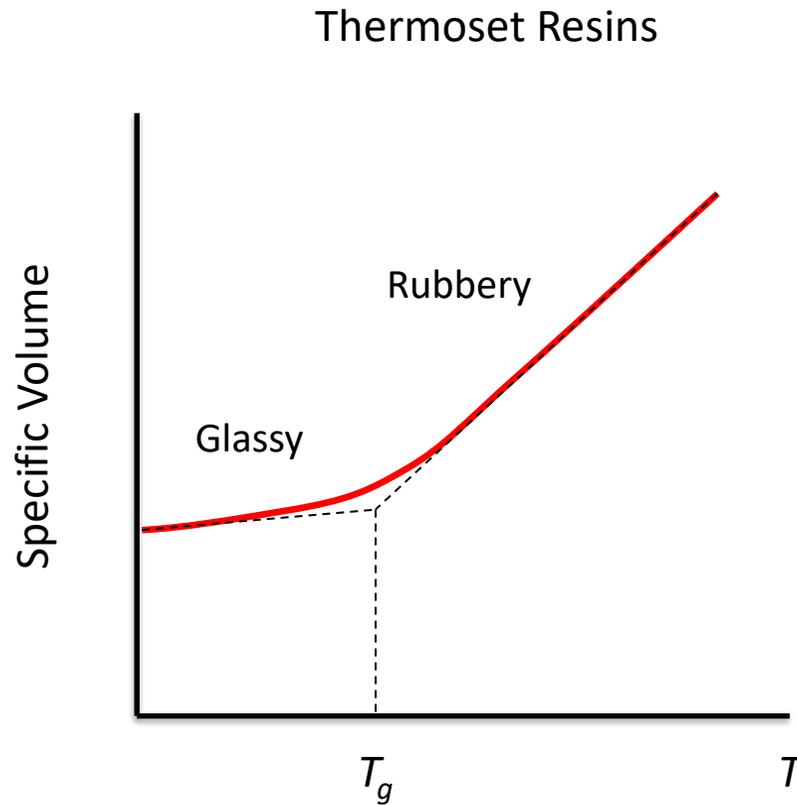


Effect of curing temperature on degree of cure

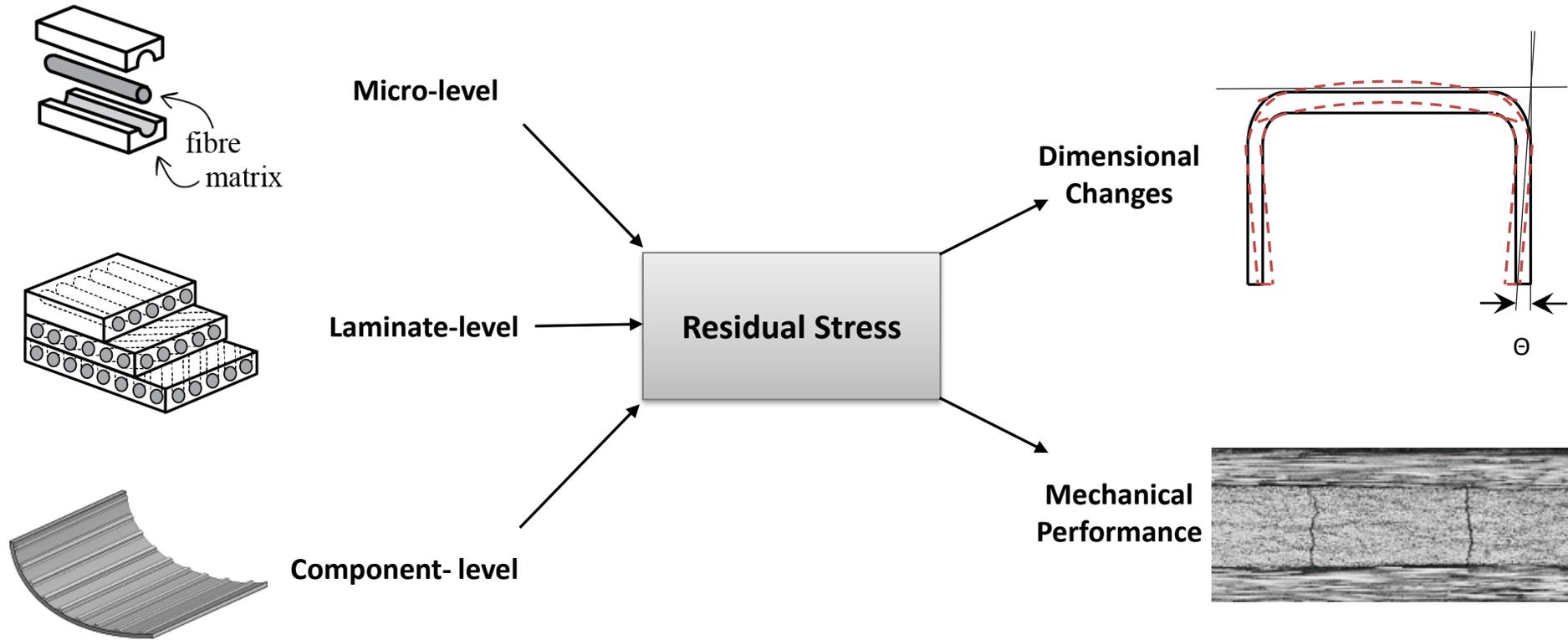


# Volume: Thermosets vs Thermoplastics

- Here, the effect of temperature on volume of thermosets and thermoplastics are compared:



# Free Strains in Composites

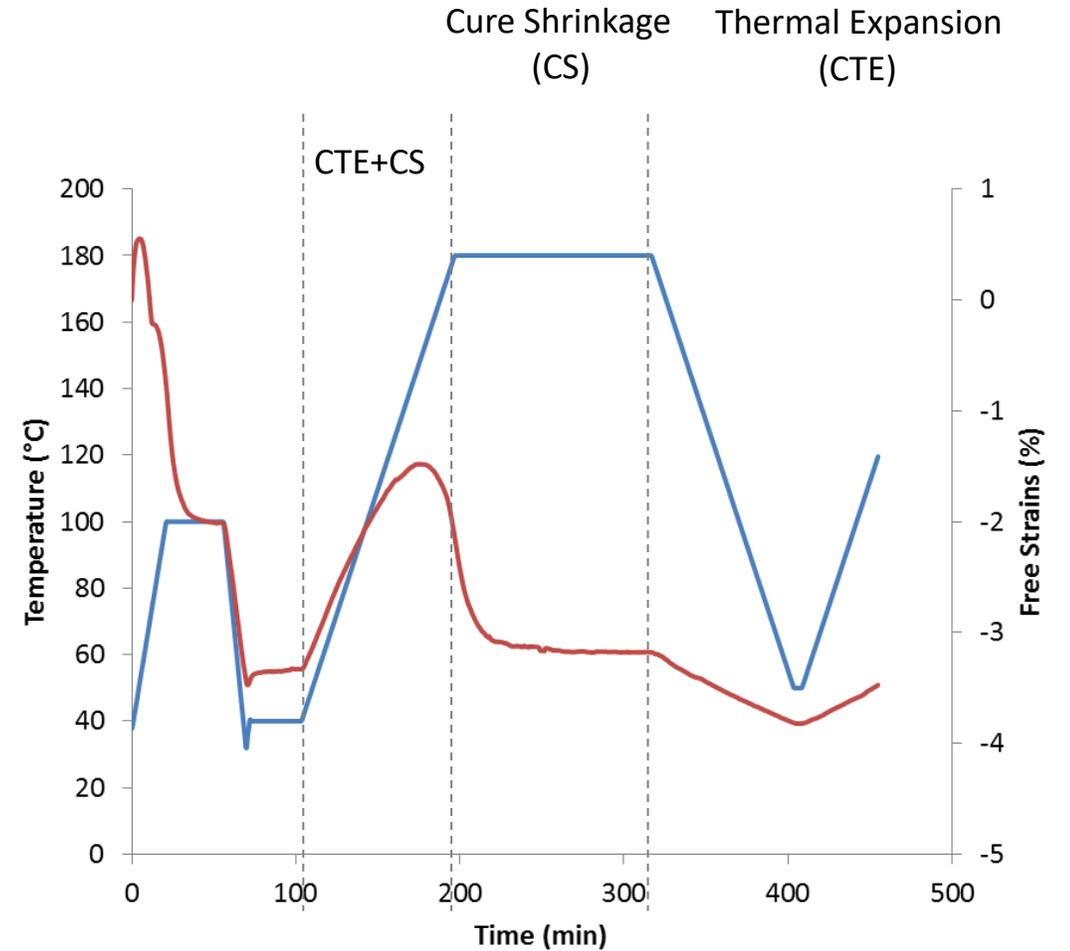
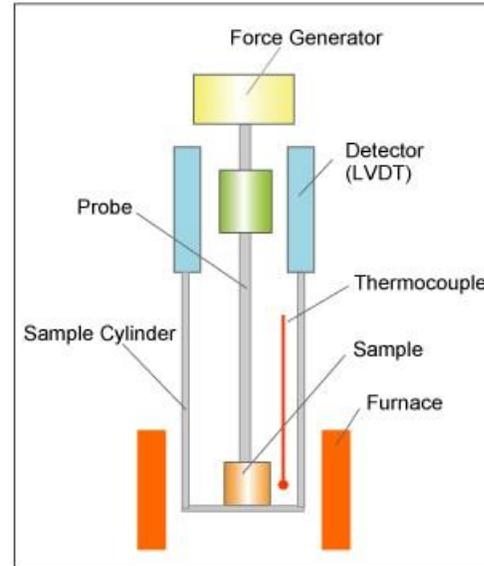


Free strains

$$\epsilon_{free} = \epsilon_{thermal} + \epsilon_{phase-change} + \epsilon_{moisture}$$

# Characterizing Volume Change During Processing

## Thermomechanical Analyzer (TMA)

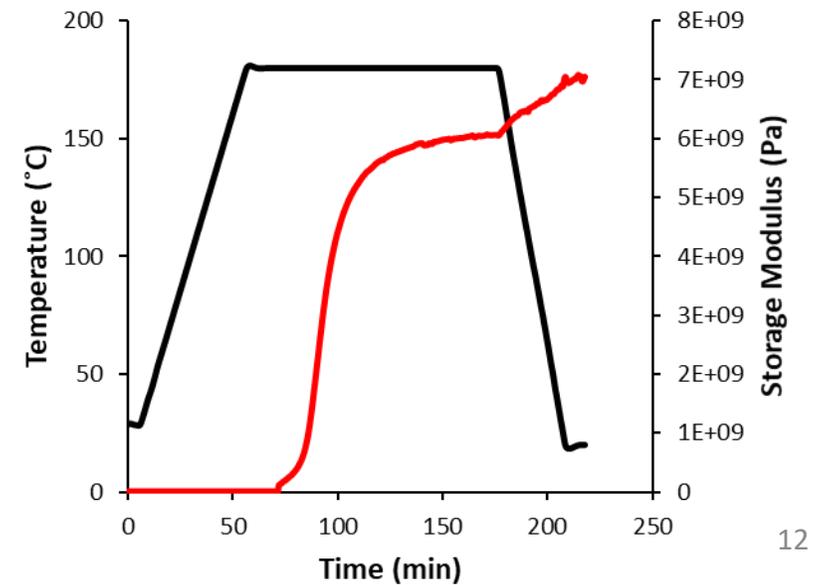
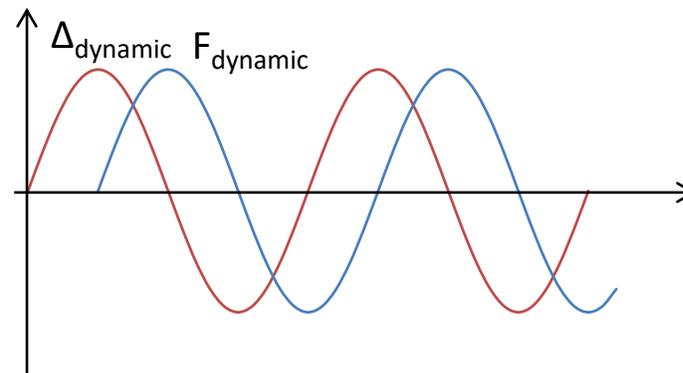
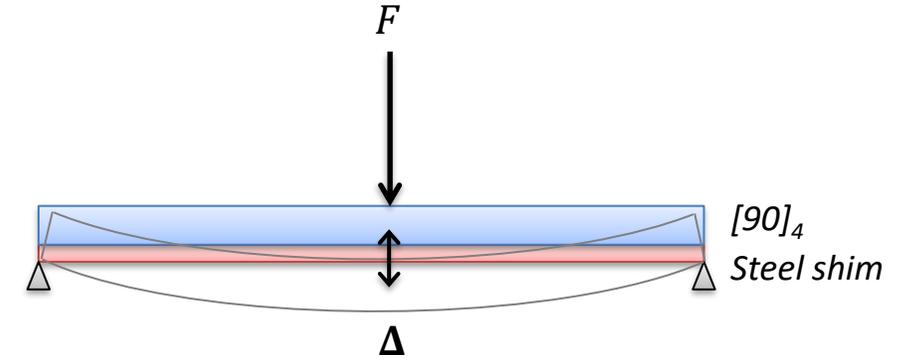


# Characterizing Evolution of Viscoelastic Properties

Dynamic Mechanical Analyzer (DMA)



Dynamic Test

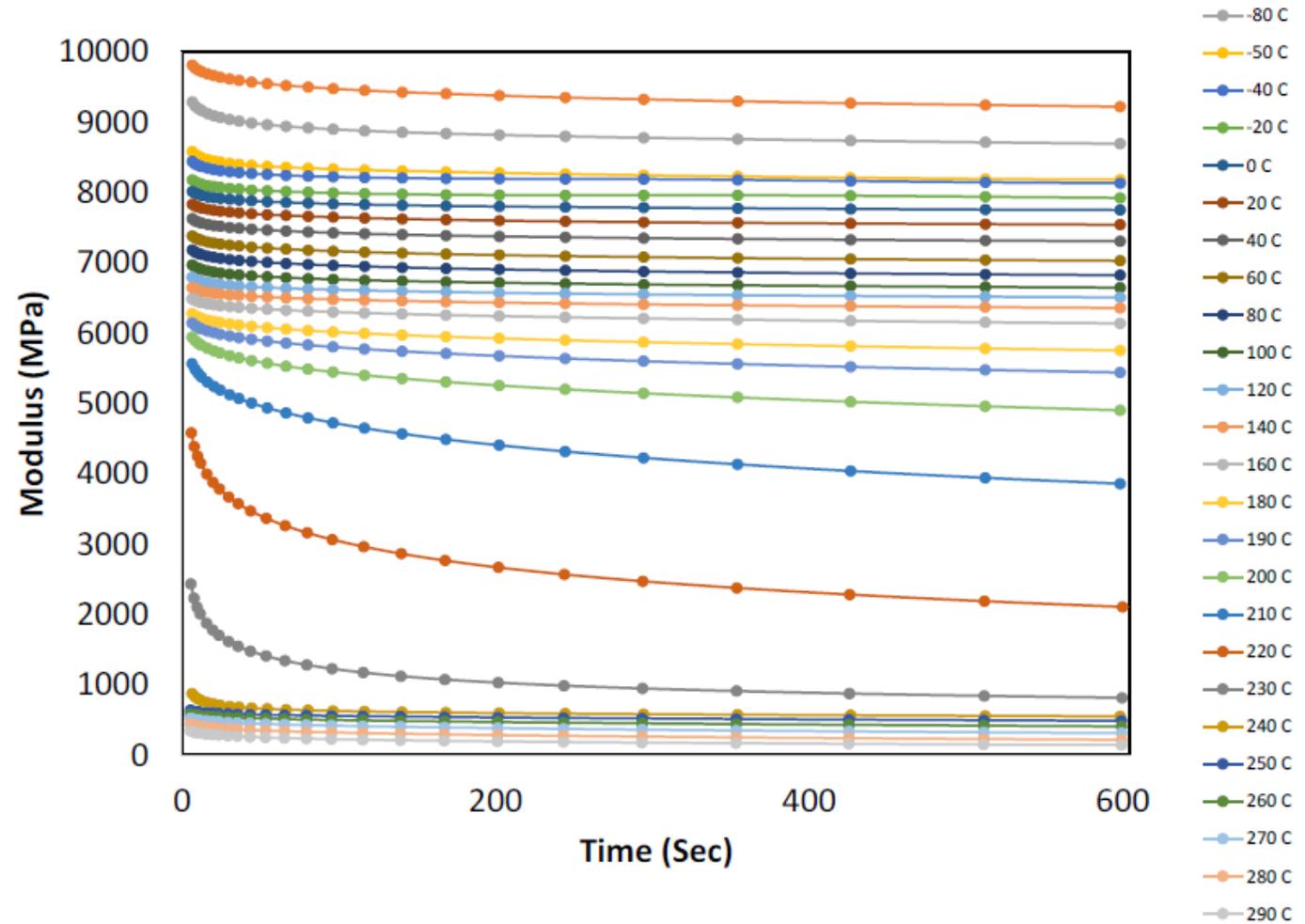


# Characterizing Evolution of Viscoelastic Properties

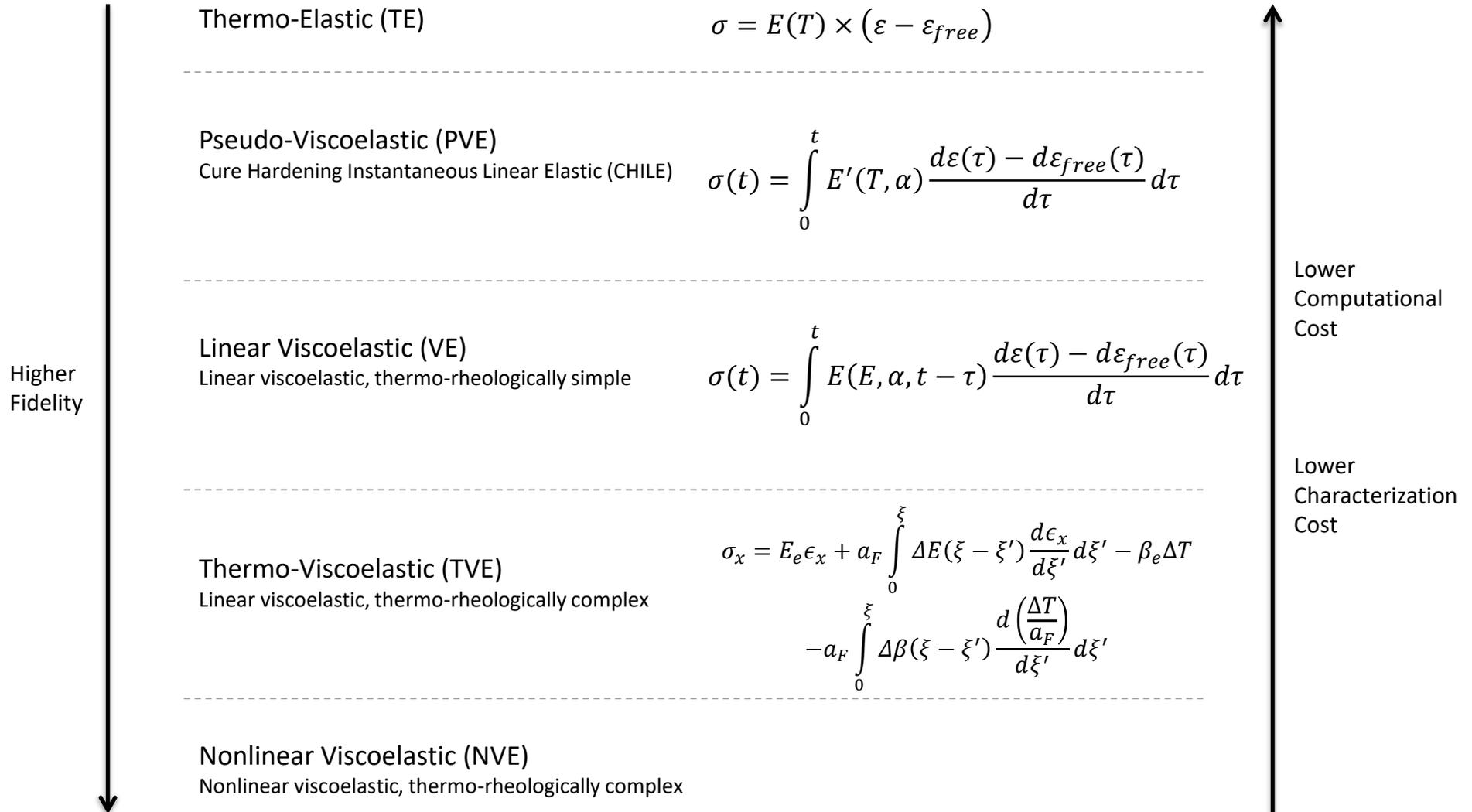
Dynamic Mechanical Analyzer (DMA)



Relaxation Tests (prepreg beam, AS4/8552)

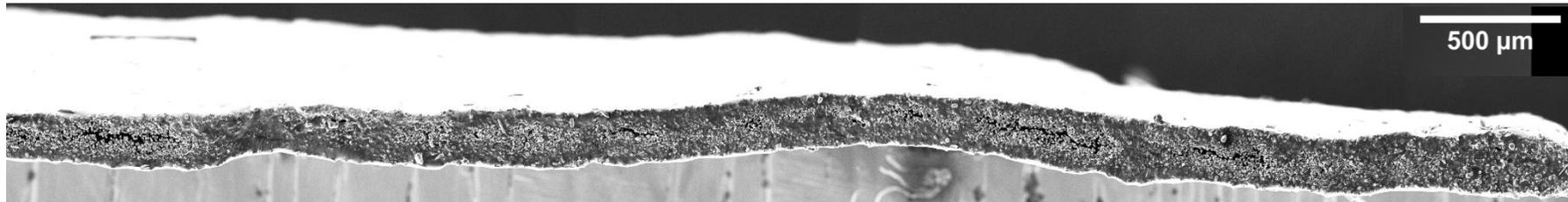


# Constitutive Models



# Initial Prepreg Condition

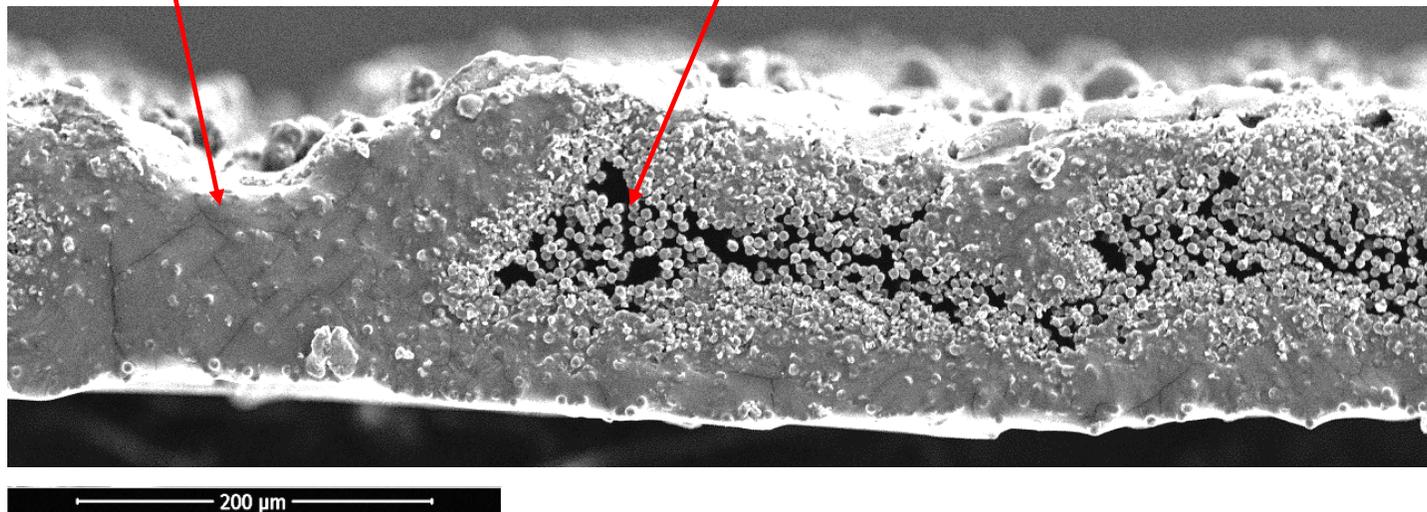
- HEXCEL AS4/8552 carbon-epoxy UD prepreg
- Made using the hot melt process



AS4/8552 cut at room temperature  
SEM Image (X200 Magnification)

Resin-rich area

Dry fibers

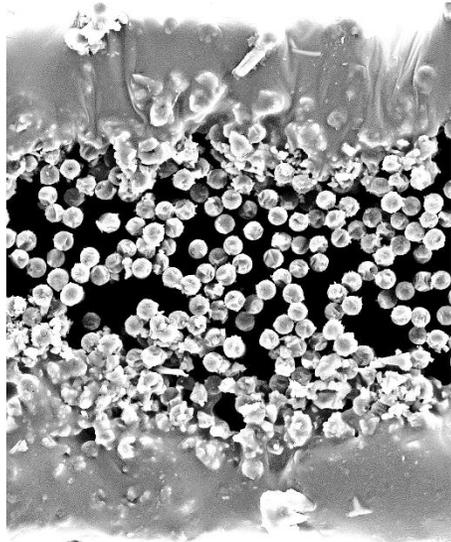


# Effect of Pressure

---

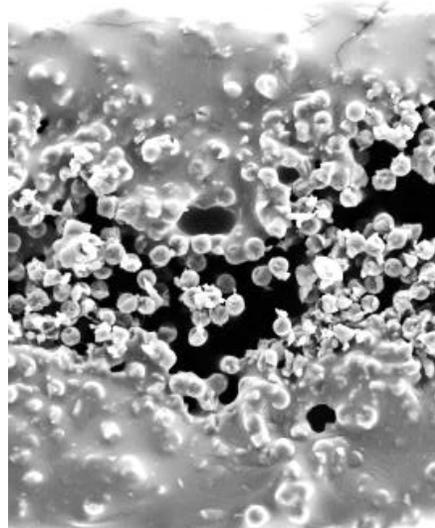
Uncured Prepreg  
(Room Conditions)

Thickness = 200 – 220  $\mu\text{m}$

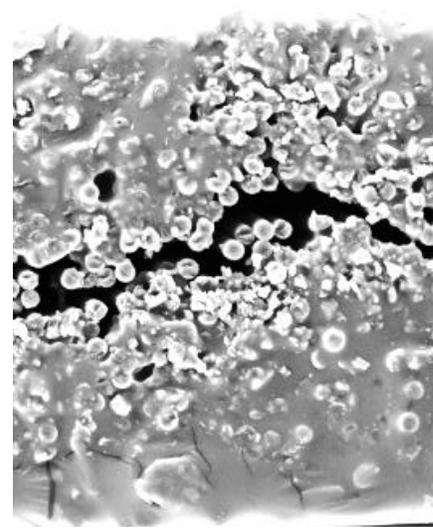


100  $\mu\text{m}$

15 min @ 40 °C (104 °F)  
(Under vacuum)

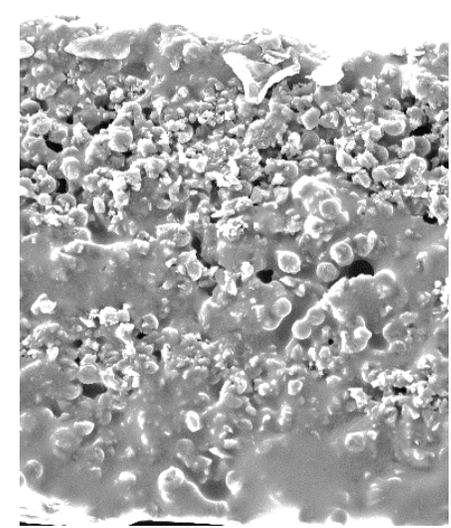


15 min @ 60 °C (140 °F)  
(Under vacuum)



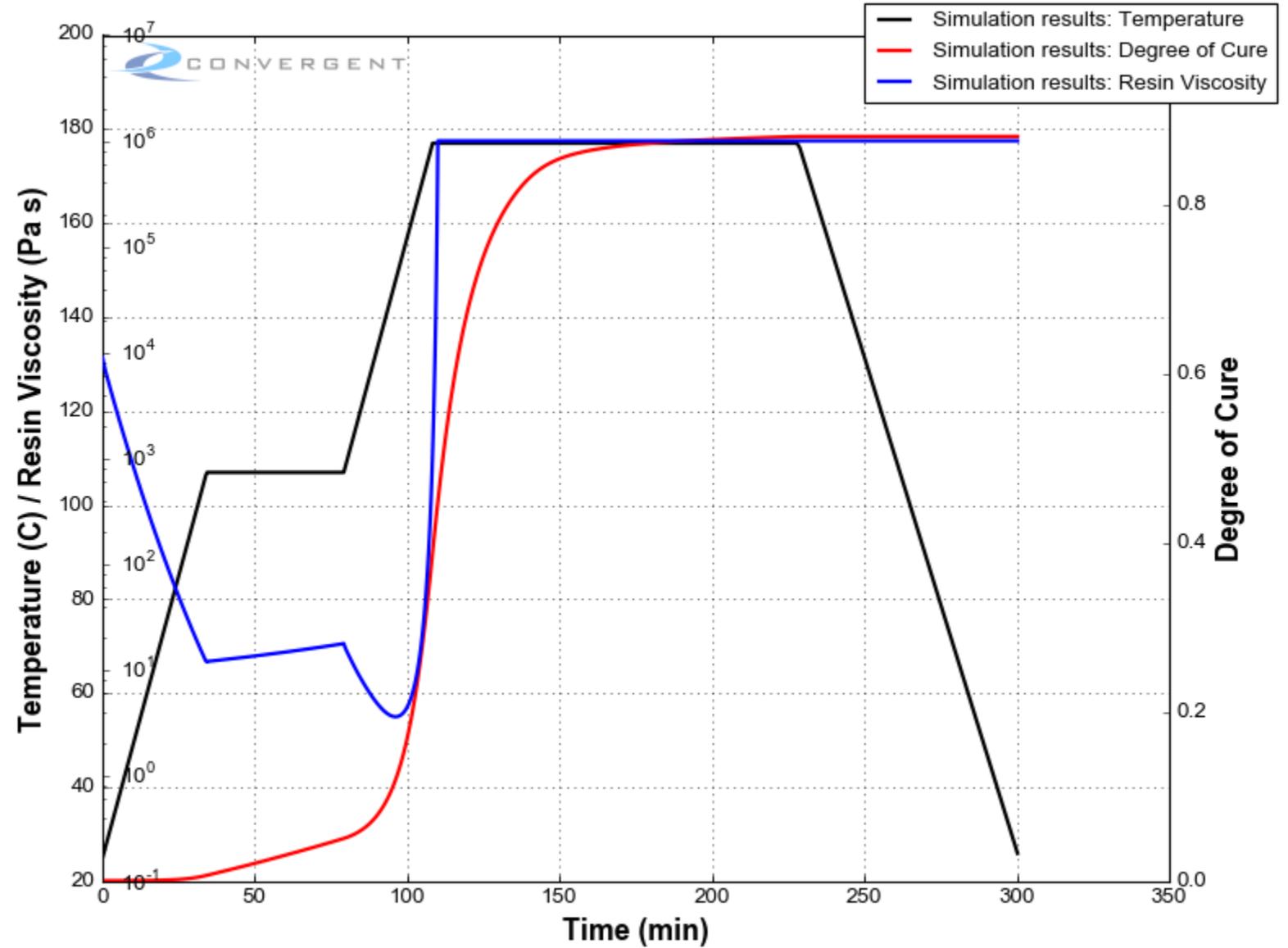
15 min at 80 °C (176 °F)  
(Under vacuum)

Thickness = 165 – 185  $\mu\text{m}$



# Characterizing Viscosity

Rheometer

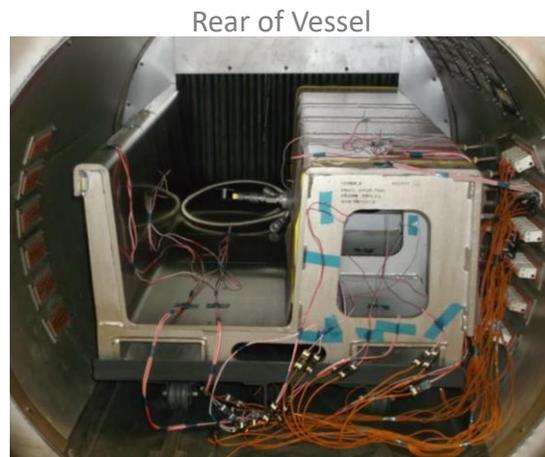


# Case Study 1: Fabricating C-Shape Parts in an Autoclave

- We want to conduct the thermo-chemical analysis for fabricating C-Shape composites parts using an autoclave:
  - C-shaped laminates @ 16, 32, and 64 ply thickness
  - Material T800H/3900-2 CFRP prepreg
  - Single hold cycle: 1.7°C/min to 2-hour hold (180 °C)

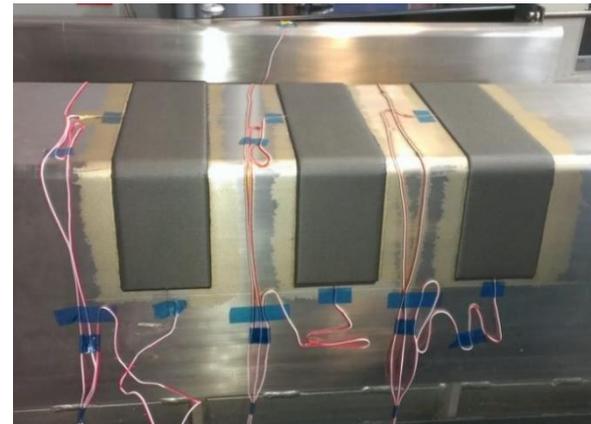


Autoclave

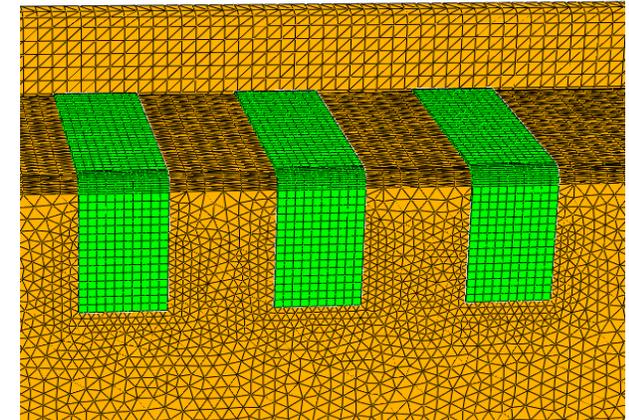


Rear of Vessel

Front (Door facing)



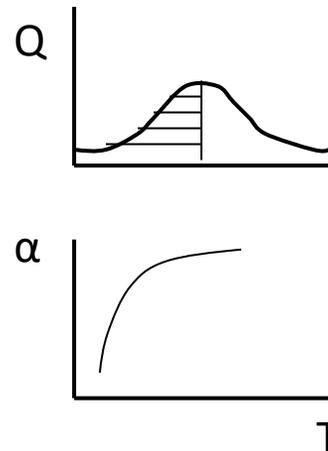
Parts on Tool



Digital Representation

# Step I: Material Characterization

- To conduct thermochemical analysis, first we need to characterize the following material properties:
  - Part cure kinetics (heat of reaction and evolution of cure)
  - Part thermal properties (thermal conductivity, specific heat capacity, density)
  - Tool thermal properties (thermal conductivity, specific heat capacity, density)



cure kinetics model

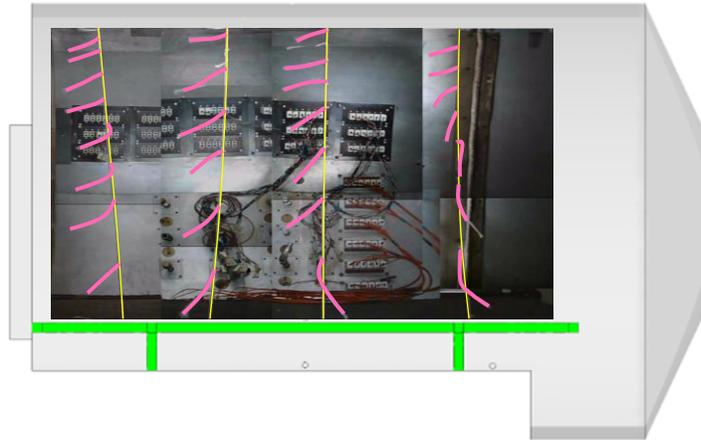
$$K = A \exp^{-E / RT}$$

$$\frac{d\alpha}{dt} = \frac{K\alpha^m(1-\alpha)^n}{1 + e^{C[\alpha - (\alpha_{c0} + \alpha_{cT}T)]}}$$

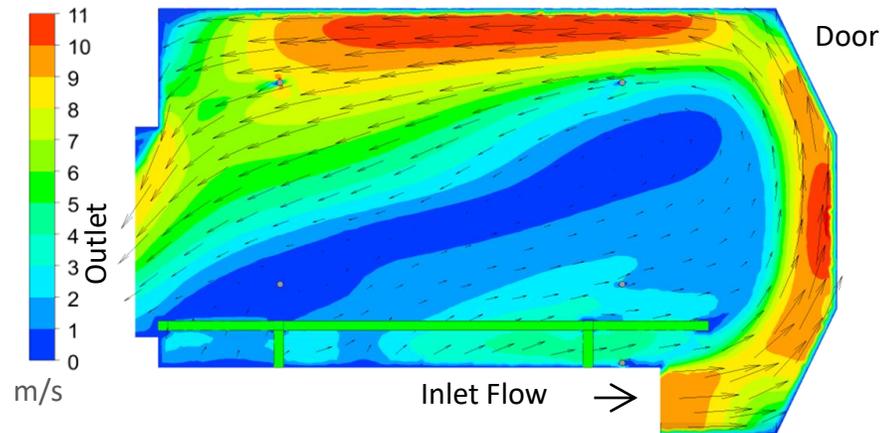
# Step II: Equipment Characterization

- We need to characterize the airflow and heat transfer coefficient in the autoclave:

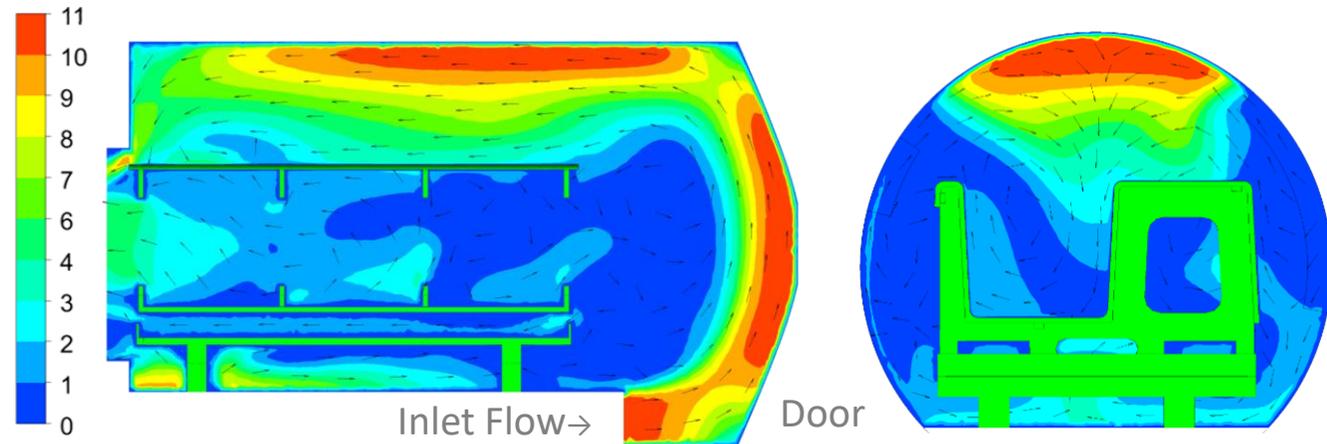
Empty Autoclave  
6.5 ft. (L) x 5 ft. (D)



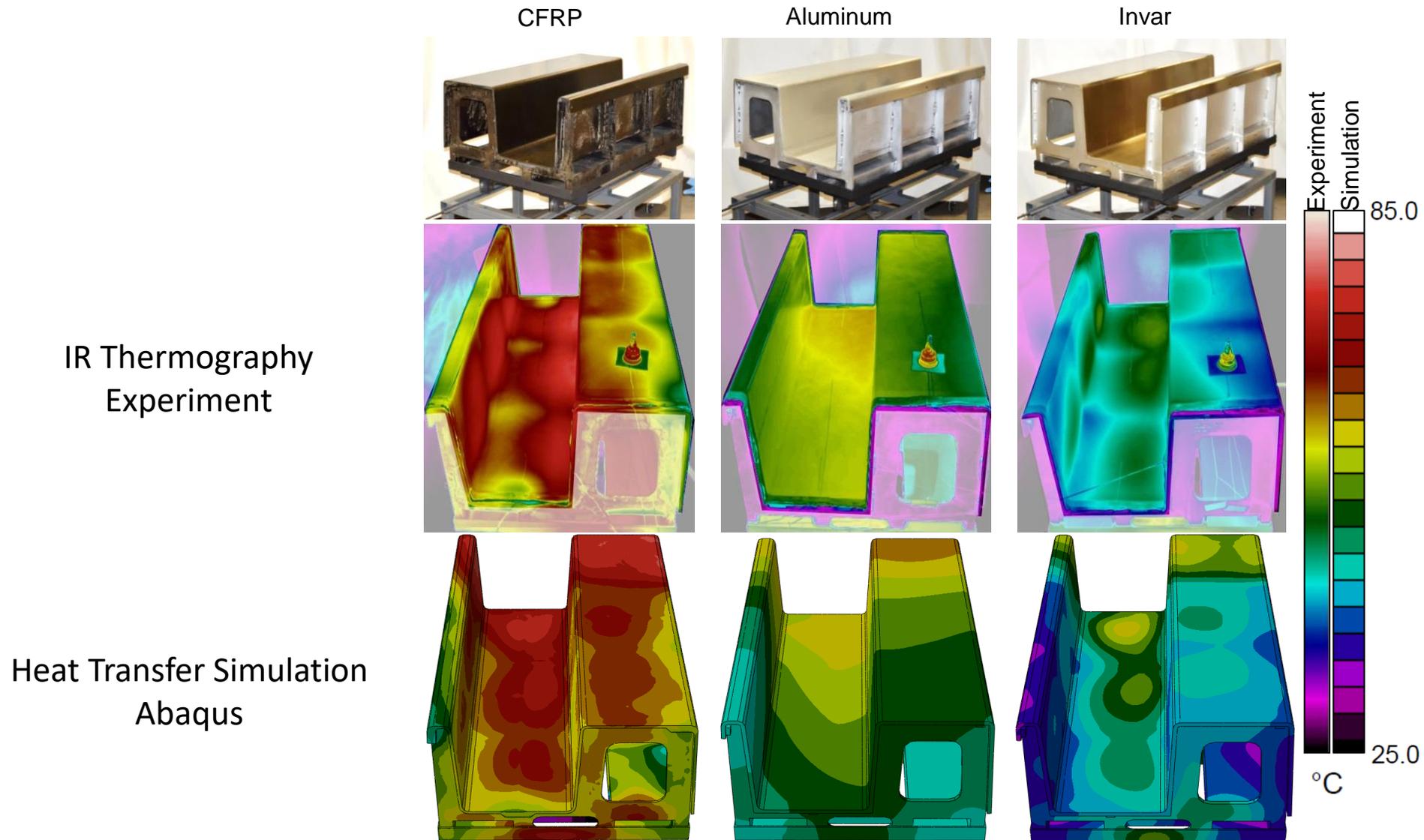
CFD, RANS-based K-epsilon turbulence  
modelling, ANSYS Fluent



With a Large Tool  
4 ft. (L) x 2.3 ft. (W) x  
1.6 ft. (H)

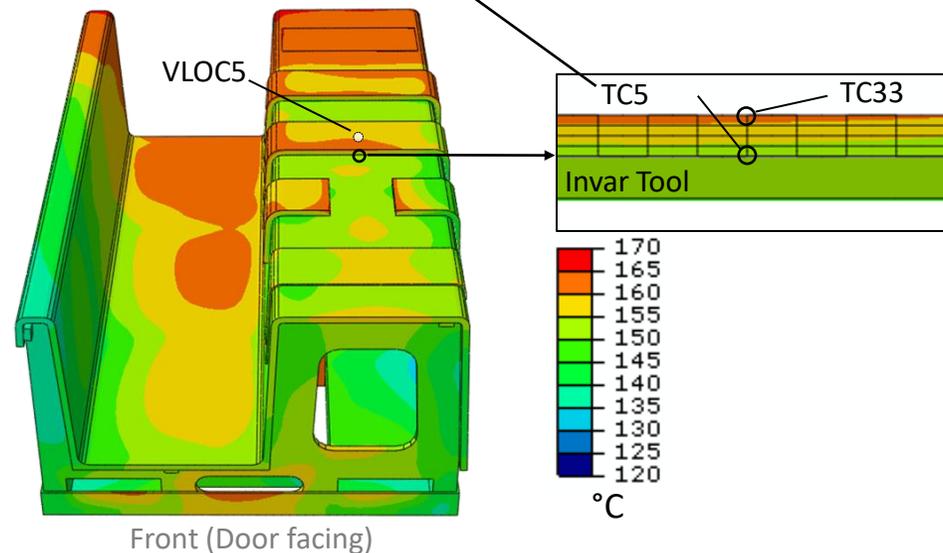
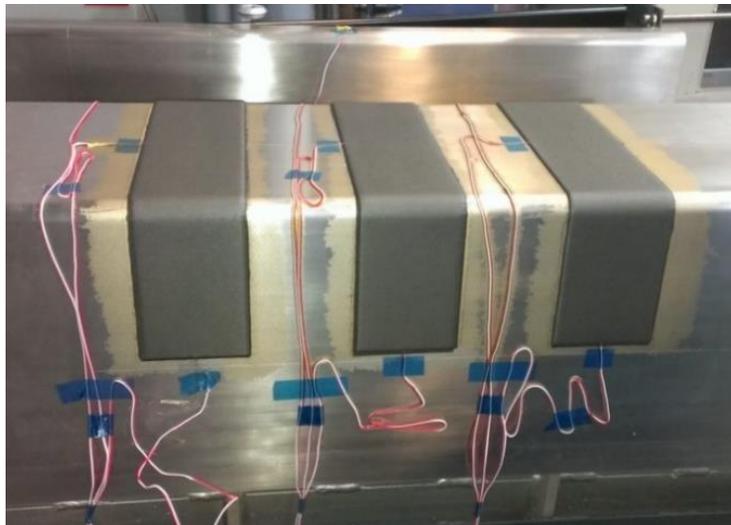
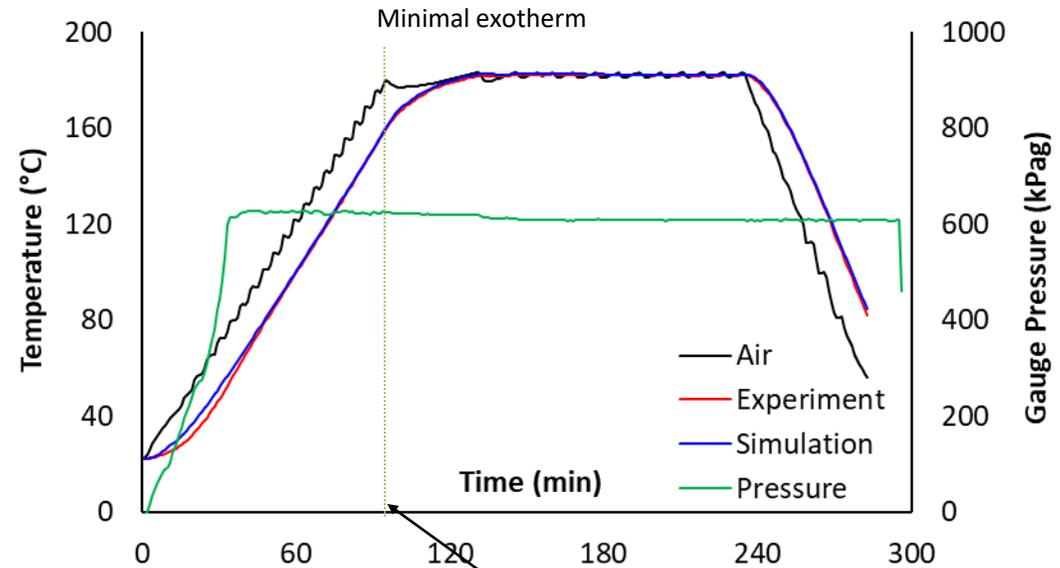


# Validation of Boundary Conditions



# Predicting Part Temperature History

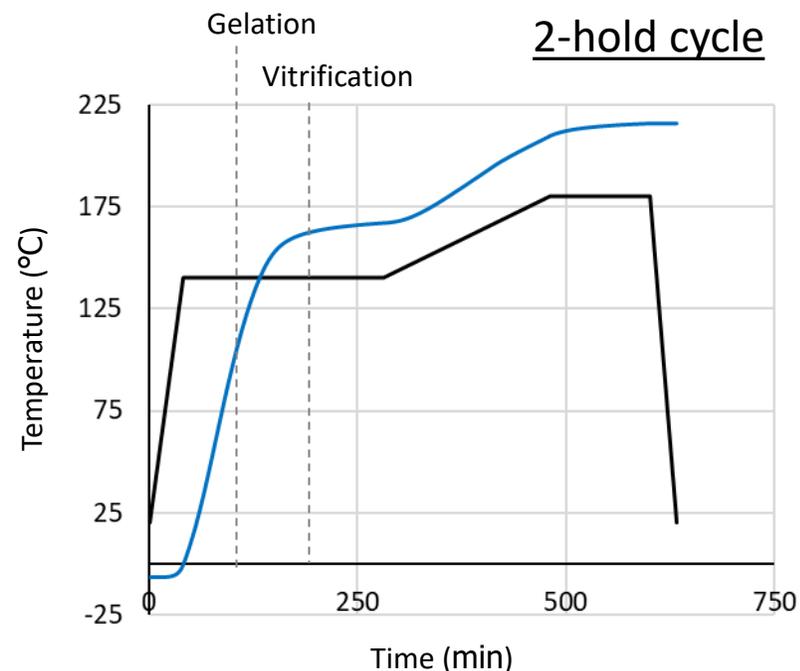
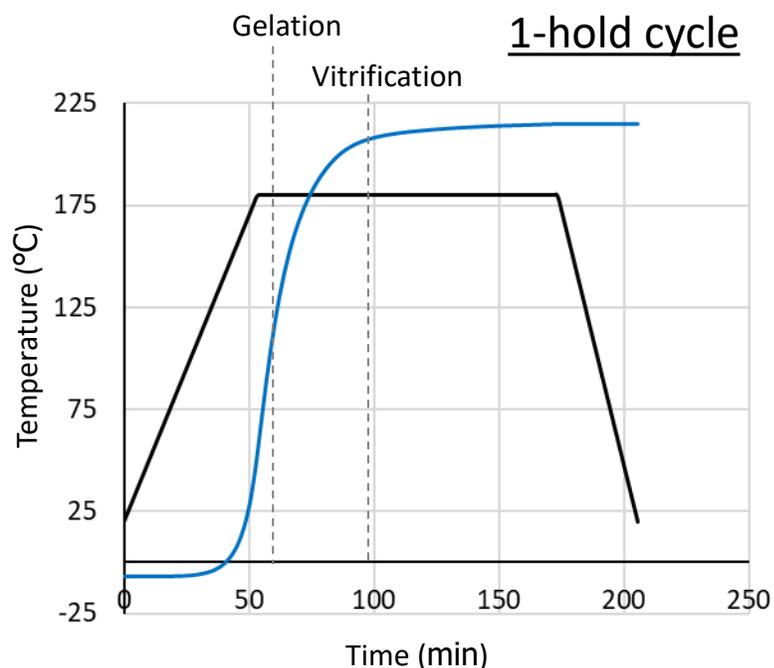
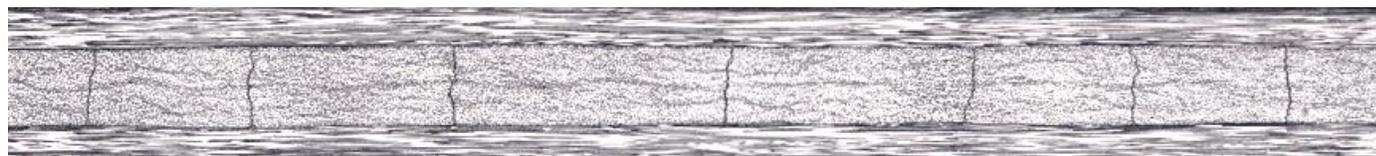
- FE Simulation was conducted using ABAQUS and implementing cure kinetics model in COMPRO
- TC data was successfully compared with FE predictions



# Case Study 2: Effect of Process Parameters on Residual Stresses

- In this study, we will investigate effect of cure cycle on residual stress and micro-crack evolution in composites:

- Material: HEXCEL AS4/8552
- Two cure cycles



— Temperature — Glass Transition Temperature (T<sub>g</sub>)

# Material Characterization

- We need to characterize the following material properties:
  - Cure kinetics
  - Free strains: Cure shrinkage and CTE
  - Viscoelastic modulus evolution

DMA



TMA

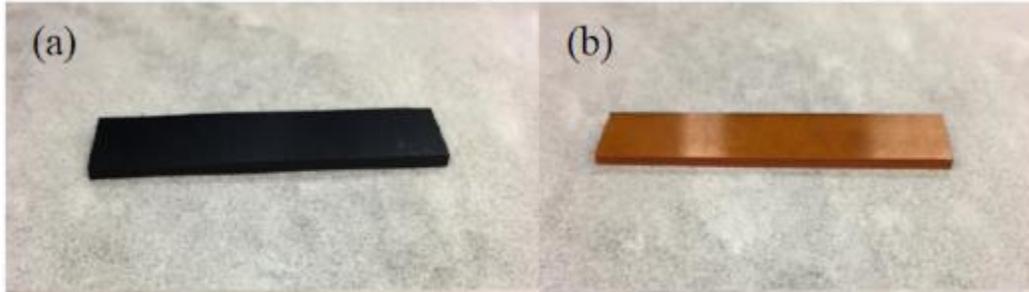


DSC



# Characterization of Viscoelastic Properties

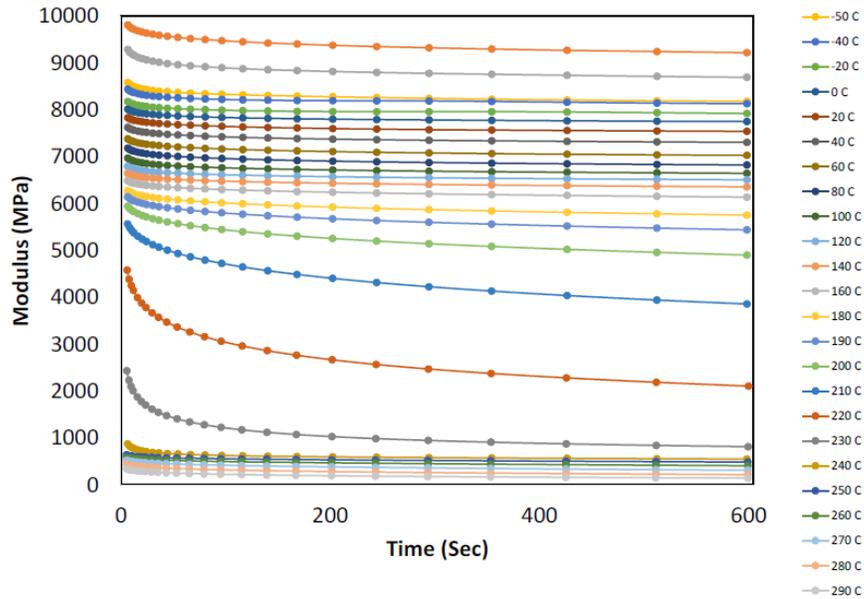
Fully Cured 8552 resin and AS4/8552 prepreg beams



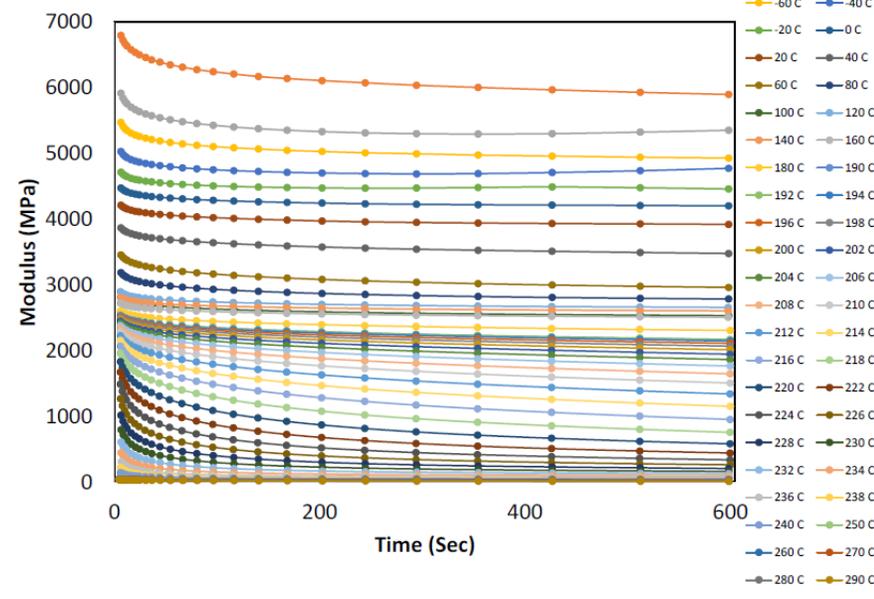
3-point bend clamp for relaxation test



Prepreg

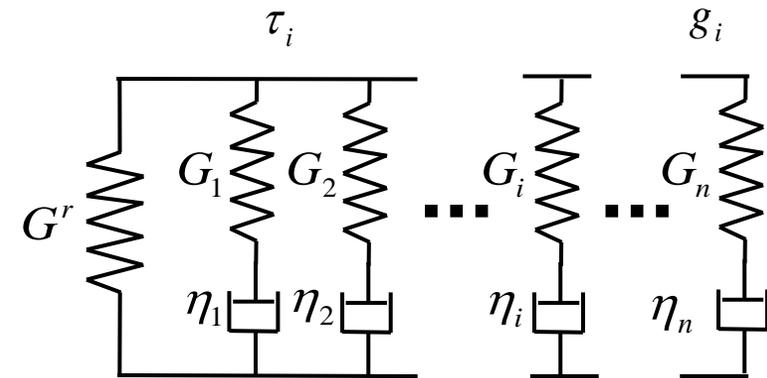
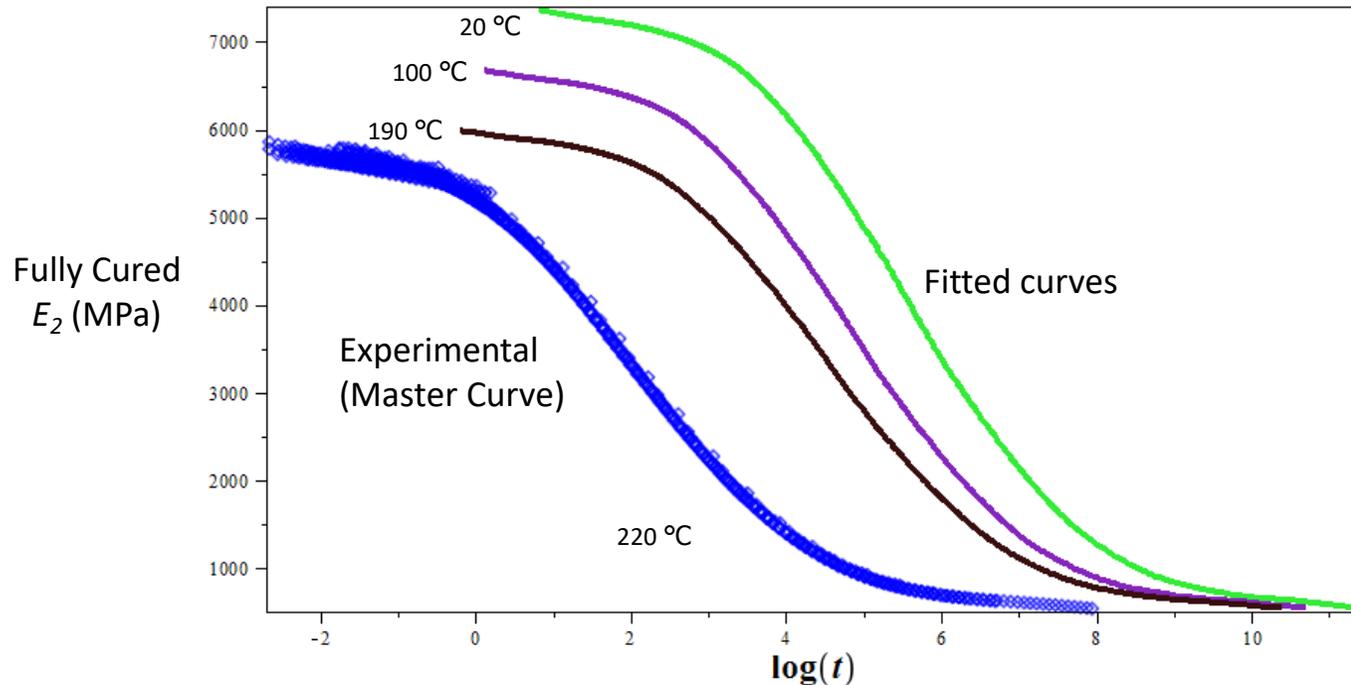


Resin



# Characterization of Viscoelastic Properties

- Generalized Maxwell Model can be characterized to describe  $G'$  (Storage Modulus),  $G''$  (Loss Modulus) and  $G$  (Relaxation Modulus)



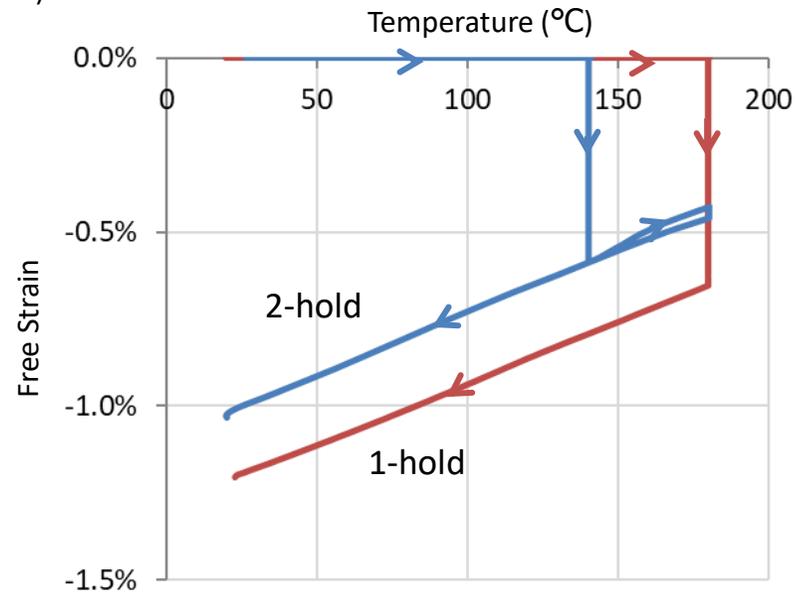
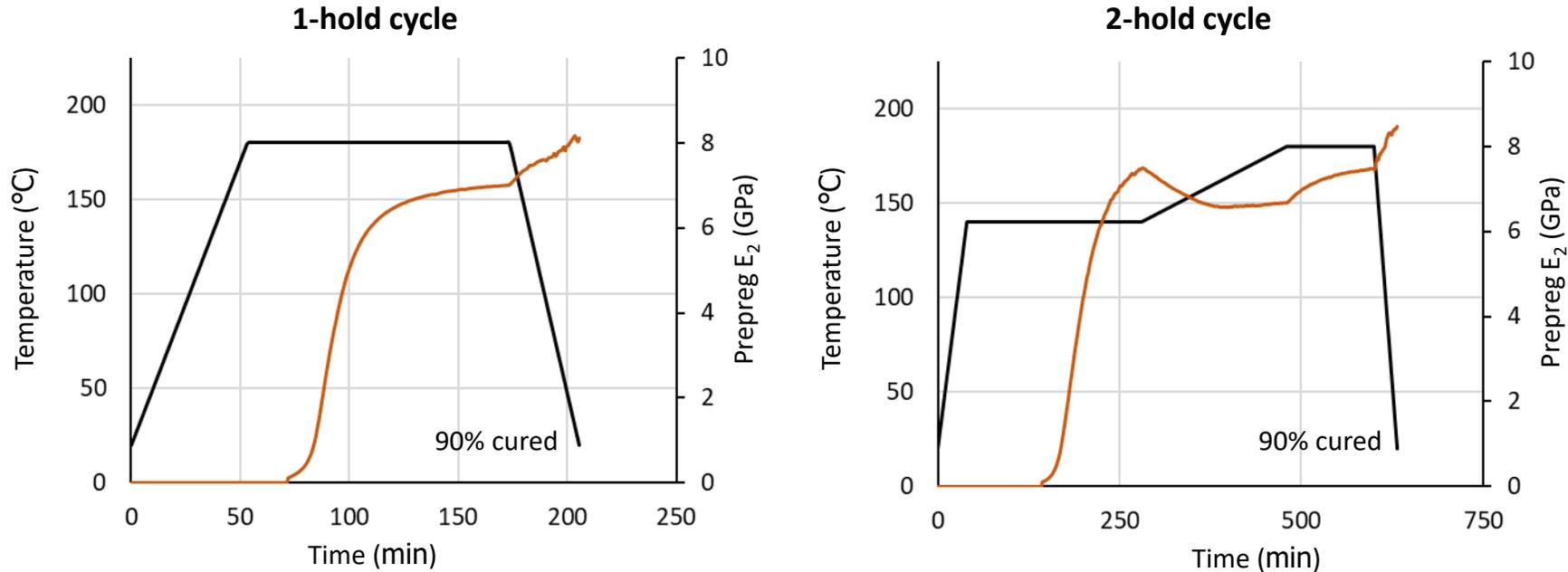
$$\tau_i = \frac{G_i}{\eta_i}$$

$$G'(\omega, T, X) = G^r(X, T) + [G''(T) - G^r(X, T)] \sum g_i \frac{\omega^2 a_{T,X}^2 \tau_i^2}{1 + \omega^2 a_{T,X}^2 \tau_i^2}$$

$$G''(\omega, T, X) = [G''(T) - G^r(X, T)] \sum g_i \frac{\omega a_{T,X} \tau_i}{1 + \omega^2 a_{T,X}^2 \tau_i^2}$$

$$G(t, T, X) = G^r(X, T) + [G''(T) - G^r(X, T)] \sum g_i e^{-(t/a_{T,X} \tau_i)}$$

# Storage Modulus and Free Strain Characterization Results



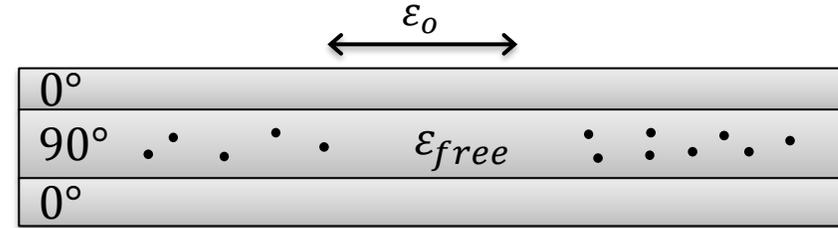
# Predicting Residual Stress in [0/90]<sub>s</sub> Laminates using Characterized Properties

$$\begin{Bmatrix} dN \\ dM \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} d\varepsilon_0 \\ d\kappa \end{Bmatrix} - \begin{Bmatrix} dN_{free} \\ dM_{free} \end{Bmatrix} \rightarrow$$

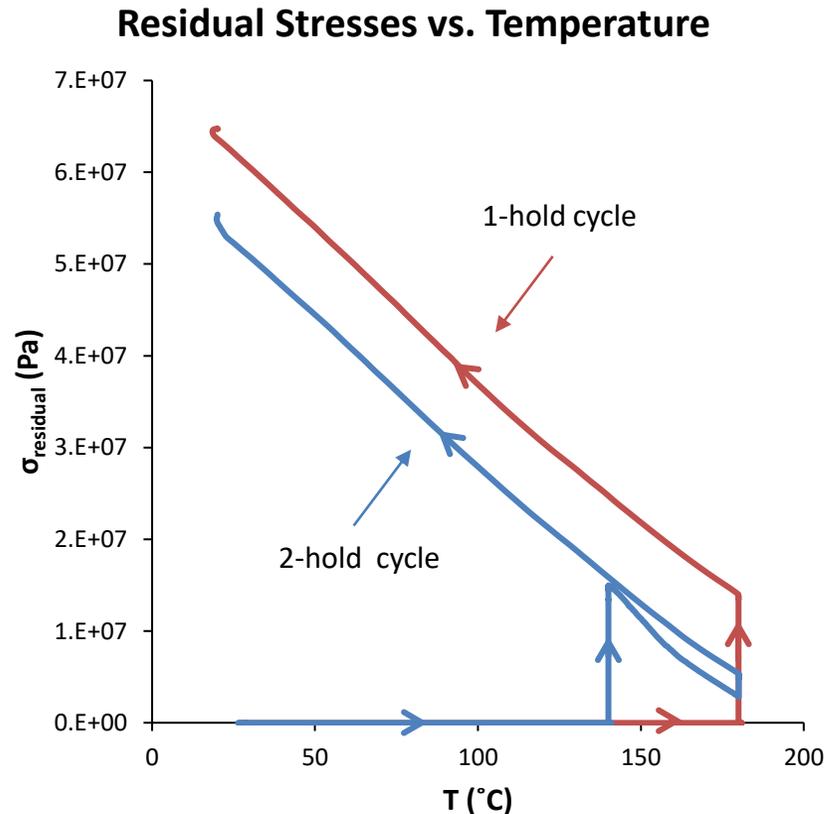
$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} A' & 0 \\ 0 & D' \end{bmatrix} \begin{Bmatrix} d\varepsilon_0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} E'_{90} t_{90} d\varepsilon_{free} \\ 0 \end{Bmatrix} \rightarrow$$

$$d\varepsilon_0 = \frac{E'_{90} t_{90}}{A'} d\varepsilon_{free} \rightarrow$$

$$\sigma_{residual} = \int d\sigma_{residual} = \frac{-E_0 t_0}{\sum E'_i t_i} \int E'_{90} d\varepsilon_{free}$$

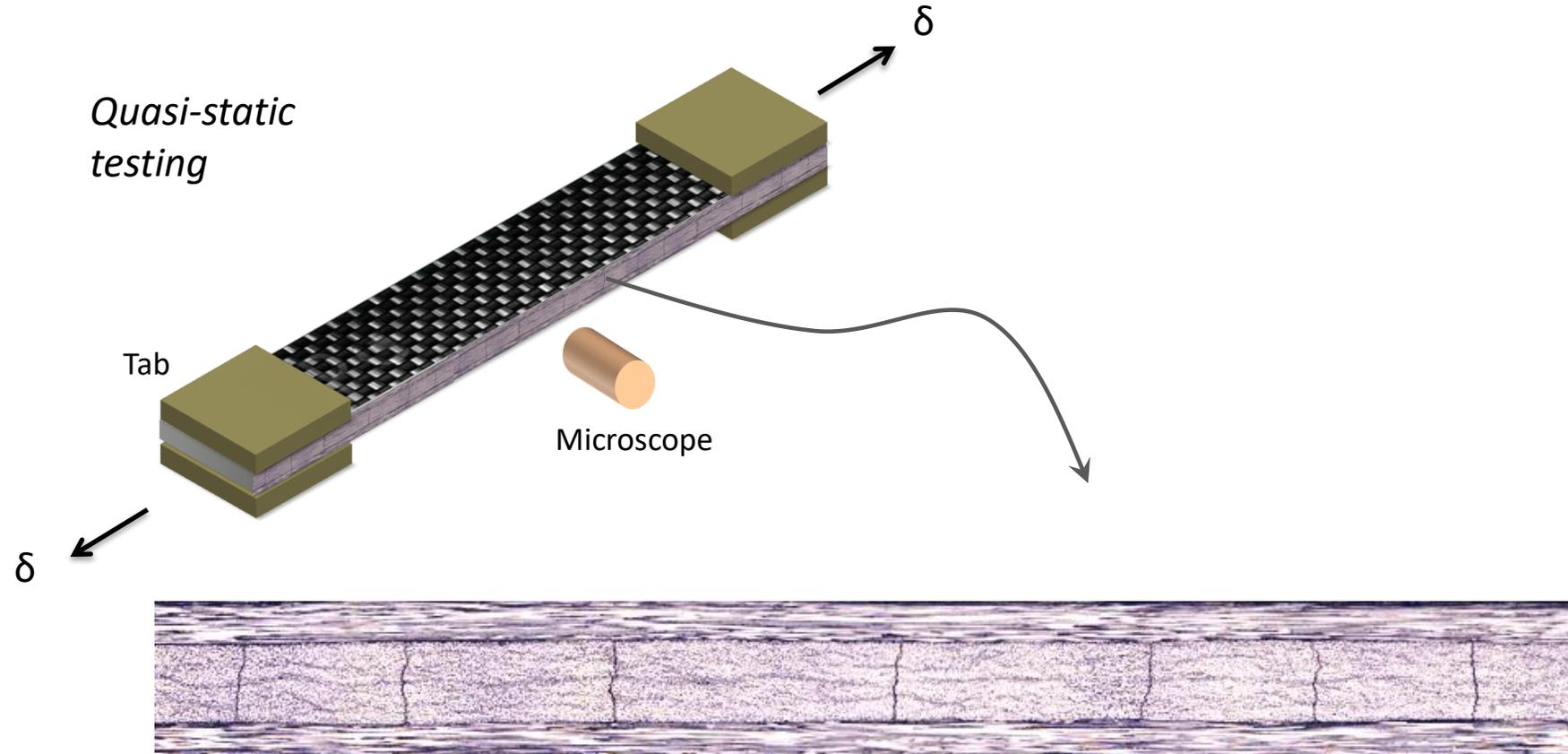


Cure Cycle	Total Strain $\varepsilon_0$ (%)	Residual Stress $\sigma_{residual}$ (MPa)
1-hold	0.648	64.8
2-hold	0.554	55.4
$\Delta$	0.09	9.4

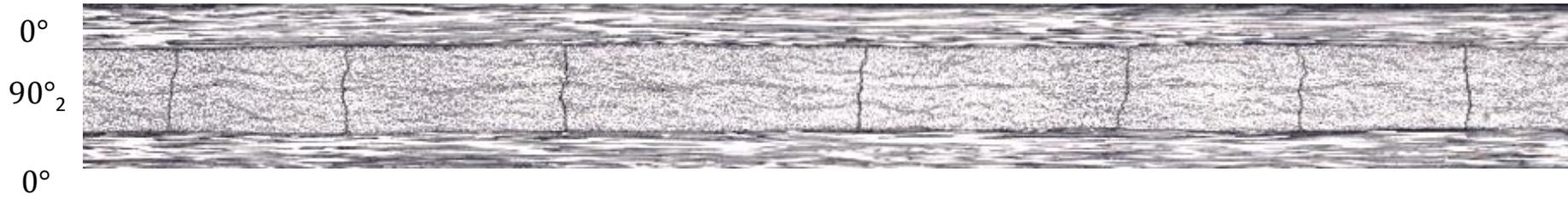


# Validation of Results

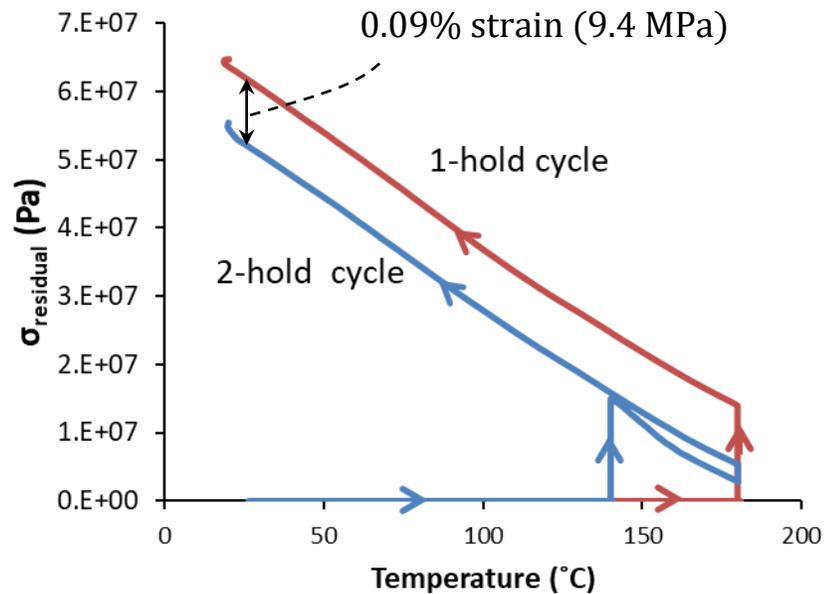
- Laminates were manufactured using the two cure cycles.
- Tensile coupons were prepared and tested while one polished edge of the coupon was monitored using a microscope.



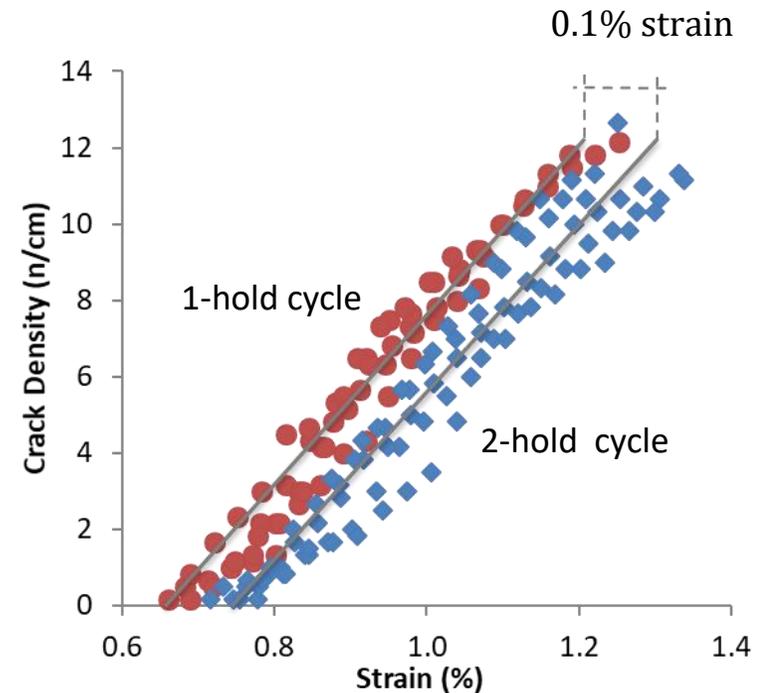
# Residual Stress in Cross-Ply Laminates



Predictions of Residual Stresses in  $[0/90]_s$  Laminates



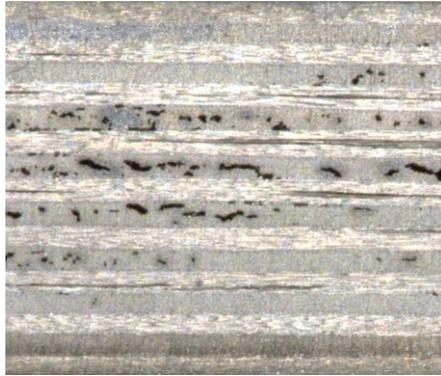
Matrix Cracking Tests on  $[0/90]_s$  Laminates



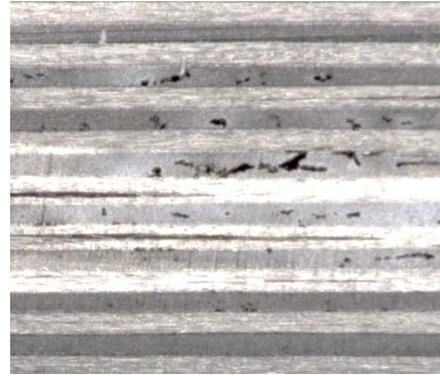
# Effect of Process Parameters on Porosity

- Effect of Cure Cycle on Porosity (Toray T800/3900-2):

Ply thickness = 0.196 mm  
( $\approx$  7% porosity)



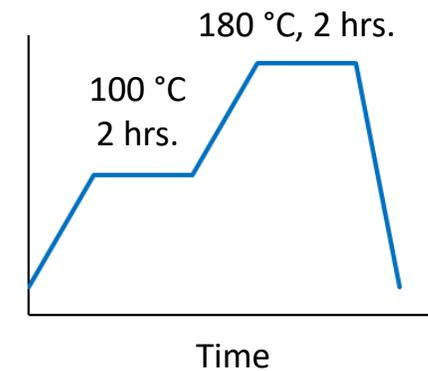
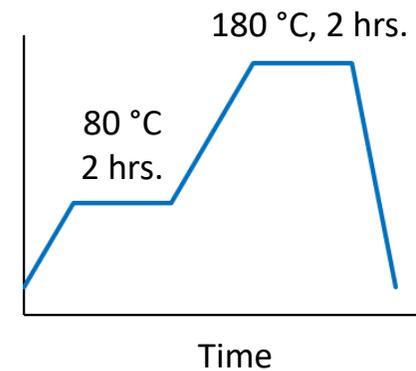
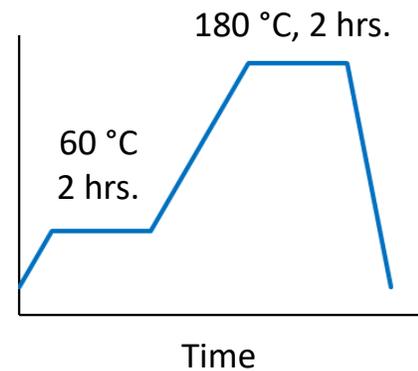
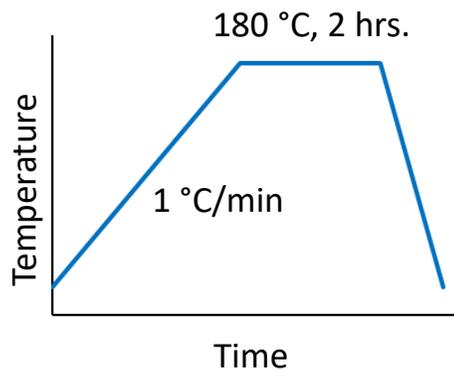
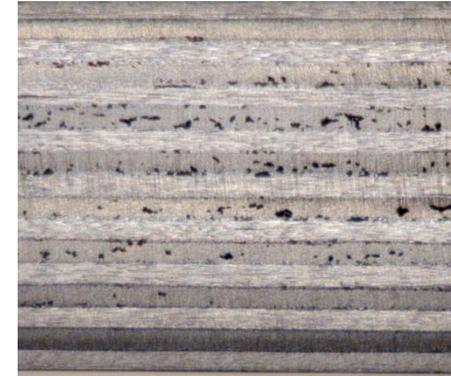
Ply thickness = 0.189 mm  
( $\approx$  3.5% porosity)



Ply thickness = 0.182 mm  
( $<$  0.5% porosity)



Ply thickness = 0.192 mm  
( $\approx$  5% porosity)



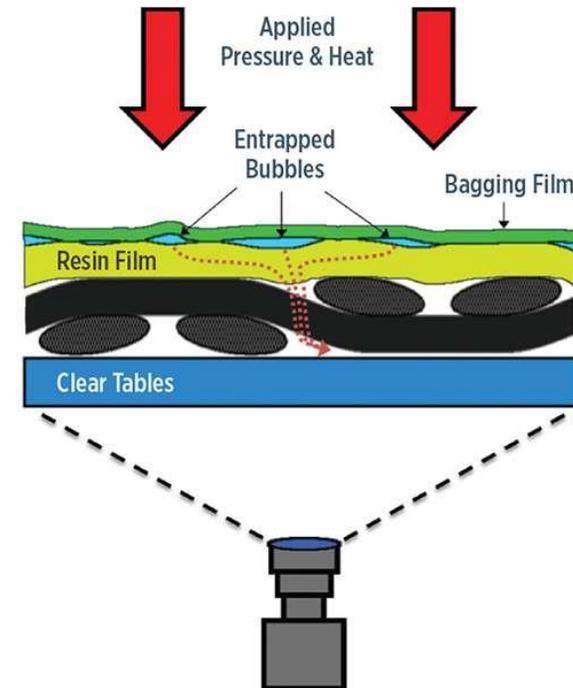
# Material Characterization

- We need to characterize the following material properties:
  - Cure kinetics
  - Resin viscosity
  - Prepreg permeability

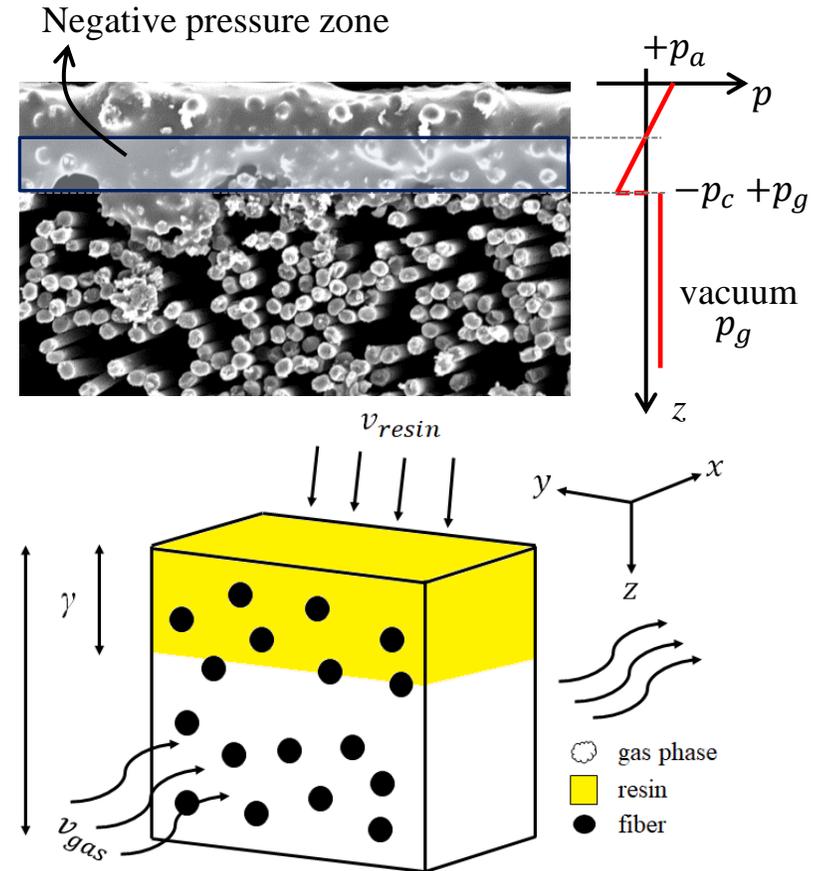
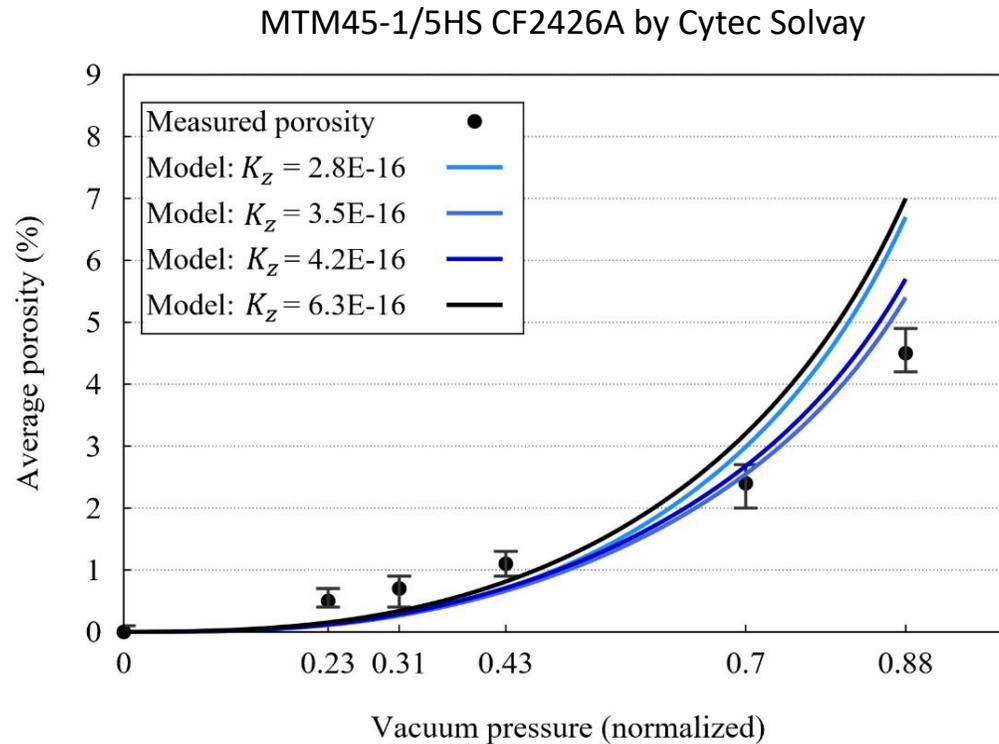
In-plane (resin infusion)



Through-thickness (prepreg)



# Coupled Gas/Resin Transport Model

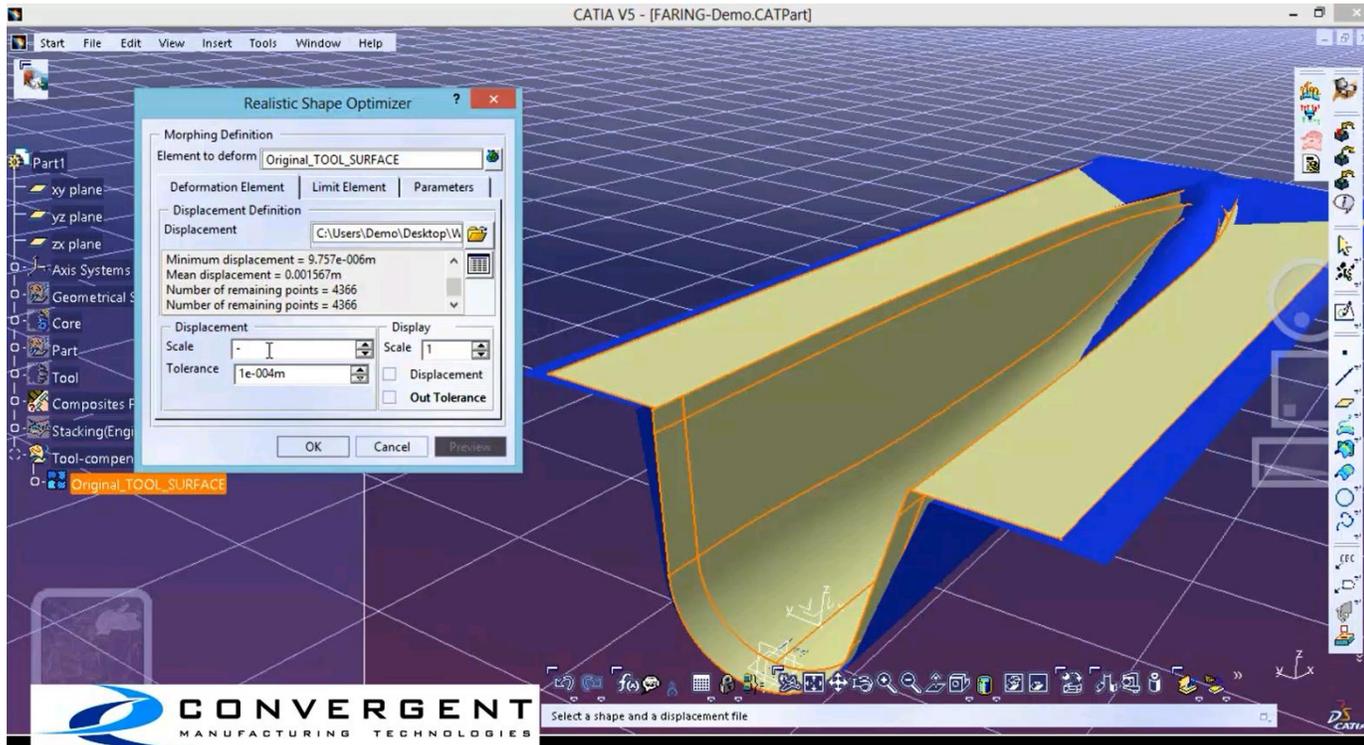


$$\begin{cases} \varphi \frac{\partial P_g}{\partial t} = \frac{K_{x0}}{\mu_g} \frac{\partial}{\partial x} \left( \left(1 + \frac{b}{P_g}\right) P_g \frac{\partial P_g}{\partial x} \right) + \frac{K_z}{h^2 \mu_r (1 - V_f)} \frac{P_g (P_a - P_g + P_c)}{(1 - \varphi)} \\ \frac{d\varphi}{dt} = - \frac{K_z}{h^2 \mu_r (1 - V_f)} \frac{P_a - P_g + P_c}{(1 - \varphi)} \end{cases}$$

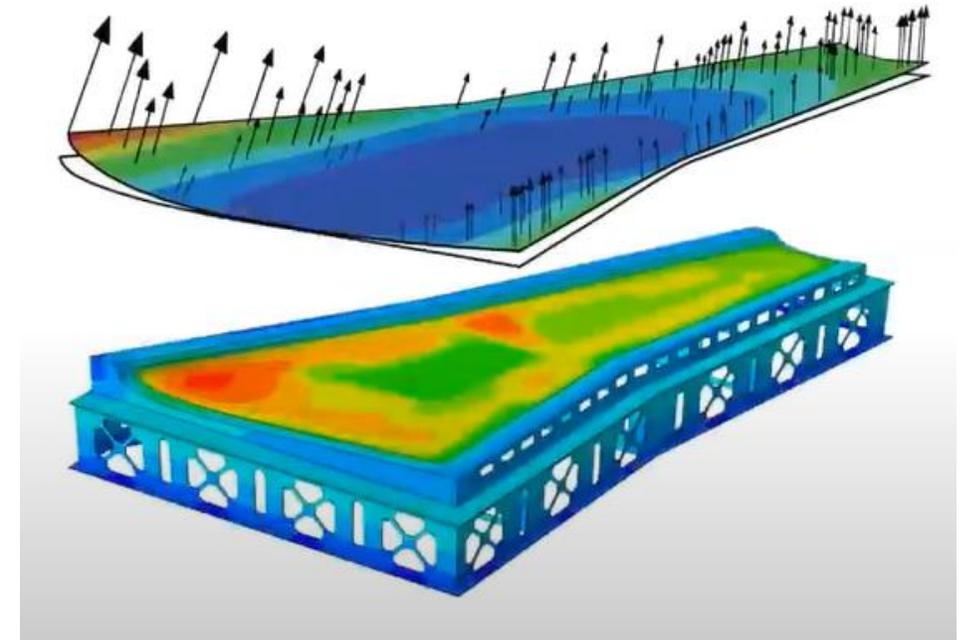
Gas flow in x direction

Resin flow in z direction

# Example: Commercial Tools for Dimensional Changes

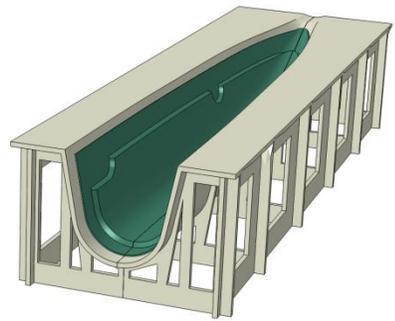


Process-Induced Deformations by COMPRO

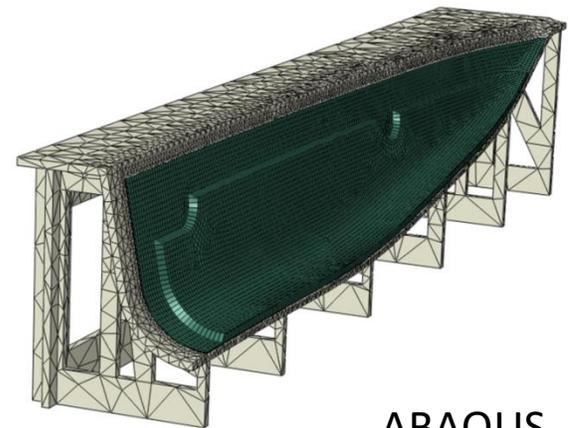


# Industry Approach

Material  
Characterization



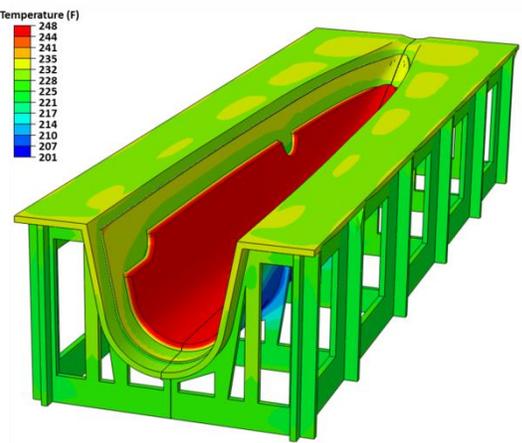
CATIA



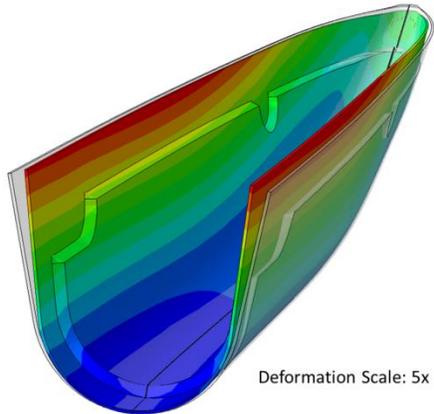
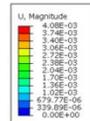
ABAQUS  
COMPRO



Thermo-  
chemical analysis



Stress-deformation analysis



Tool Geometry  
Compensation